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A STUDY OF
A LOW-INTERMODULATION
TRIODE PLATE MIXER

A THESIS

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A STUDY OF A LOW-INTERMODULATION TRIODE PLATE MIXER

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PREFACE

This thesis is the culmination of a research problem bent upon demonstrating the practicality of a triode plate mixer for electronic frequency changing in applications for which the production of intermodulation interference by the frequency changer must be held to a very small value.

The work as presented here is not of a highly complex mathematical nature. Rather, the effort has been to utilize a combination of mathematics and physical reasoning in a fashion acceptable to the practicing engineer. Where suitable authoritative points of departure have been available from the literature, these have been used without extensive proof of rigor; yet where the duplication of analytical development appearing in the literature has seemed useful for purposes of clarity and/or continuity, such work is given. Available, unclassified literature devoted to the specific problem of this paper has been found to be meager. Actually, however, the character of the device examined here borders on that of similar devices for other purposes about which much has been written. To the end that the gap between the familiar and the unfamiliar should be bridged is this thesis justified.

Beyond what is suggested by the title, the scope of this thesis is limited to considerations of unmodulated continuous waves or of double-sideband, normal-carrier, amplitude-modulated waves. The intermodulation interference considered is that which is manifested as interference on a desired channel because of the coexistence, specifically, of two or more undesired r-f signals, whether these signals are modulated or not. Thus, extraneous products brought about by the intermodulation of various sideband components or of various audio components of the desired signal are not of primary interest here, although the conclusions reached will apply equally well to these cases.

A completely and rigorously detailed mathematical treatment of all possible facets of the subject would constitute a monumental task quite beyond the scope of this thesis. However, the overall approach has been greatly simplified by the advancing of certain arguments and practical assumptions designed to circumvent the necessity for a great deal of complex mathematics. In recognition that this approach is likely to lead to some loss of accuracy and completeness, experimental evidence is presented to demonstrate to some degree the extent to which the results can be relied upon in normal practice. In general, however, the nature of testing of the type required here is such that possible experimental errors are likely to be comparable to, or (excepting careful precautions) even to overshadow, the probable deficiencies in the developed theory. Notably, intermodulation measurements are quite likely to be subject, among other things, to intermodulation resultants generated within the test equipment itself.

The important things, however, are that the formulas and methods suggested offer practical working tools and that a confidence is established in the worth of a triode plate mixer where low-intermodulation frequency changing is required.

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Floyd P. Holder

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ABSTRACT

The principal purpose of this thesis is to develop design considerations and parameters for a practical, low-interference plate mixer. As a preliminary to an attack on the main problem, a qualitative comparison of various mixers is made with respect to probable freedom from interference.

Both the comparison and the design development utilize a simplified power-series representation of anode current. Since this expansion describes a continuous, physically realizable situation, it is assumed convergent and the first- through the fourth-degree terms are assumed to approximate satisfactorily the entire series. A detailed examination of these terms leads to design criteria for the low-interference plate mixer considered as a maximum-voltage generator.

This study shows the triode plate mixer to be superior to other known types of mixers with respect to rejection of interference. Relative to the equivalent grid-mixer the class-A_1 plate mixer has approximately $\frac{1}{\mu}$ as much conversion gain but can adequately handle signals μ times as large. The improvement in interference rejection afforded by the plate mixer as compared to the equivalent grid mixer is proportional to a positive integral power of μ, the exponent being 2 for fourth-degree interference and proportionately greater than this for still higher degrees.

The conversion gain of the plate mixer operated in class A_1,

as recommended, is found to be independent of μ. It is also found that while the amount of fourth-degree mixing relative to second-degree mixing is independent of the signal-voltage amplitude, the relative amount of fourth-degree interference is dependent upon this amplitude. The dependence is such as to imply a desirability for keeping signal levels low.

Several design stipulations are developed for the plate mixer used as a maximum-voltage device. The tube selected should be of a high-plate-voltage type having a high μ and a large grid base. It should be operated at maximum allowable plate voltage and maximum local-oscillator swing of the grid within class-A_1 limits. The plate load should approximate a resistance which, compared to the tube plate resistance, is small at radio frequencies, large at intermediate frequencies, and zero at all other frequencies including zero. The operating point should be adjusted to that at which greatest intermodulation rejection occurs, this point having been approximately located from theory.

Experiment bears out the trend of the theory but indicates the development to be optimistic with respect to conversion gain and pessimistic with regard to output-amplitude stability as a function of oscillator voltage.

Further work on the plate mixer is recommended to disclose its characteristics as a maximum-power device, as a device operating at or beyond the limits of class A_1, and as a device employing some type of feedback. A noise study is also recommended.

CHAPTER I

INTRODUCTION

The ever-increasing demand for and utilization of frequency allocations within the limits of the usable radio-frequency spectrum have acutely increased the problem of interference with service on one "channel" or spectral segment caused by the presence of signals on one or more other channels. Not only is the interference per se highly objectionable, often completely "jamming" or blanketing the conveyance of information in a given case, but also the relatively wide interchannel frequency spacing necessitated by the effort to alleviate the interference problem has resulted in an inefficient use of available spectrum space.

Many an artifice has been employed in an attempt to circumvent the interference difficulty. One such device is time-sharing of the same spectrum space by two or more facilities on a basis of nonsimultaneous use. For many applications, however, the aggregate of these schemes is either inadequate or impractical, and a more technical approach becomes essential. Especially acute are situations which arise when a particular service must crowd into an allocated frequency band a large number of simultaneously operative stations which are located in close physical proximity to each other and which must be capable of consistent exchange of information with remote, possibly weak, stations. In addition, the signal strength received from, or at, a remote station may well be varying between wide limits, possibly at a rapid rate. In addition, if either the transmitting station or the receiving station or both are in motion, as frequently happens, then the total situation is made more difficult, from a technical viewpoint, by the frequency shifting produced by the Doppler effect.

Such a combination of conditions can and does occur, for example, in the case of a government air force control-center installation. Here a large number of relatively powerful stations, perhaps located in the same building and operating with frequency spacings limited by the bounds of an allocated frequency band, may well be in simultaneous but separate communication with aircraft flying in groups or individually at unpredictable speeds at indefinite distances and assorted directions through uncertain and possibly closely spaced antenna-pattern lobes. The set of circumstances thus envisioned is not so much pessimistic as it is realistic, and while it is already a frequent occurrence, it will undoubtedly be even more commonly encountered as time passes.

Certainly not all the effects mentioned are normally classed as interference, but it is wise to take cognizance of all of them in any thinking toward reducing, eliminating or avoiding interference. In addition to natural and man-made noise and to two or more stations occupying the same spectrum space, the principal types of interference to electrical conveyance of information may be broadly classed as "spurious," "cross-modulation" and "intermodulation." The word "spurious" is used here with some license to describe all responses or emissions other than those desired. In this thesis, harmonics and subharmonics are included in this classification unless otherwise specified. The remaining two terms are intended to conform to the concise definitions given for them by I.R.E. Standards,[1] cross-modulation being defined as "modulation of a desired signal by an undesired signal," and intermodulation being defined as "the modulation of the components of a complex wave by each other in a nonlinear system." Modulation, in turn, is defined[2] by the I.R.E. as "The process or result of the process whereby some characteristic of one wave is varied in accordance with another wave." In conformance with this definition of modulation there are, for a sinusoidal carrier, only two major categories of modulation, all others being subclassifications of one of these or of a composite of the two. These two categories are amplitude modulation and angle modulation. Unless otherwise specified, amplitude modulation is the type being thought of throughout this paper; however, the implications and conclusions reached regarding the plate mixer apply in sense to either classification.

At this point let it be stipulated that, unless otherwise specified, the signals considered here and throughout this thesis are carriers at radio and/or intermediate frequencies (as opposed to audio), capable of, and intended for, carrying modulation but not necessarily bearing it in a given case.

It is seen from the foregoing definitions that cross-modulation may be interpreted as a special case of intermodulation. In order to avoid ambiguity in this paper, more restricted definitions will be given for these two terms. First, let it be assumed that the transfer characteristic of a mixer is adequately represented by a power series containing a finite number of terms. This series would have the form

$$i = a_o + a_1 e + a_2 e^2 + a_3 e^3 + \dots + a_n e^n \tag{1}$$

where

i = output current, and

e = sum of desired signal(s), local-oscillator signal(s), and any number of undesired signals.

For a mixer represented by the foregoing, every output frequency can be expressed in terms of the input frequencies by a polynomial of the form

$$f_o = \pm m f_d \pm n f_{l.o.} \pm c_1 f_1 \pm c_2 f_2 \pm \dots \pm c_q f_q \tag{2}$$

where

f_o = a particular output frequency from the mixer,

f_d = desired-signal carrier frequency,

$f_{l.o}$ = local-oscillator frequency,

$f_1, f_2, \dots f_q$ = undesired-signal carrier frequencies, and

$m, n, c_1, c_2, \dots c_q$ = positive integers.

If only one of the integral coefficients is nonzero and this one is unity, the resulting output frequency is the same as the corresponding input-signal frequency. If only one of the integral coefficients is nonzero and this one is different from unity, the resulting output frequency is classed as a harmonic of the corresponding input frequency. If $m = 1$ and $n = 1$ and all the c's are zero, the resulting output frequency for the usual case is either the intermediate frequency or the image, depending upon the equipment design and whether the algebraic signs preceding the mf_d and the $nf_{l.o.}$ are alike or opposite. If any two or more of the c's are not zero, then the output component is said to be <u>intermodulation</u>.

Now, if all the c's are zero but $m \neq 0$ and $n \neq 0$, two types of output components (discounting images) are included: desired signal plus distortion, and <u>cross-modulation</u>. No mathematical distinction between the two can be made on the basis of frequencies. However, the component is distinguishable as cross-modulation if the amplitude coefficient of the term in question contains a factor which is some function (integral power) of the amplitude coefficient of one of the undesired signals. Thus, if the component under discussion is represented by an expression of the type

$$A_d^{\,r} A_{l.o.}^{s} A_k^{\,x} \cos 2\pi t(\pm\, mf_d \pm nf_{l.o.}),$$

then cross-modulation by the $k^{\underline{th}}$ undesired signal exists. It is apparent that the entire term vanishes if the interfering signal is "turned off" so that A_k becomes zero. It is also evident that if A_k is a variable quantity--say, because of the undesired signal being a modulated wave--the resultant will also be a function of the variations. If only desired signal plus distortion are present in the mixer output component being represented, then the factor $A_k^{\,x}$ may be replaced by unity and the expression will have the form

$$A_d^{\,r} A_{l.o.}^{s} \cos 2\pi t(\pm\, mf_d \pm nf_{l.o.}).$$

It is important to note that the production of either intermodulation or cross-modulation is inherently dependent upon the application of the component signals to a common nonlinear device. It is also worthwhile to observe that in the presence of a desired signal, intermodulation and cross-modulation interference may well exist simultaneously, it being difficult in ordinary operation to differentiate between the two phenomena.

Considerations in this paper are limited largely to the single category, intermodulation, and that primarily in a single device, namely the plate mixer. However, while it is not always true, a cure for intermodulation is usually tantamount to a cure for cross-modulation, since the two effects stem from the same origin, the nonlinearities of some device or component. Certainly that which reduces the nonlinearities from a dynamic viewpoint likewise reduces both cross-modulation and intermodulation. The same type of improvement also has its effect upon the generation of undesired harmonics.

Preparatory, now, to the study of the plate mixer, the following analogy is appropriate. The relationship of the plate mixer to other types of mixers may be likened to the relationship, for amplitude modulation, of the plate-modulated amplifier to other common means of accomplishing modulation, as, for instance, the grid-modulated amplifier. It has long been known that, among the simpler and most popular modulation schemes, plate modulation has offered quite an appreciable advantage from a standpoint of minimizing the production of amplitude distortion. On the other hand, various embodiments of grid-modulation have presented a distinct advantage from a standpoint of producing a desired degree of modulation with a minimum requirement for modulator power. Similarly, the plate mixer provides low-intermodulation (and cross-modulation) mixing at the expense of having less conversion gain than the comparable grid mixer, as will be seen later.

It will be helpful to keep the foregoing analogy in mind throughout the reading of the remainder of this paper.

CHAPTER II

INTERMODULATION COMPARISON OF VARIOUS MIXERS FOR VHF - UHF OPERATION

A consideration of the problems of r-f cross-modulation and intermodulation interference in a receiver quickly leads to the possibility that a frequency changer, or "mixer" as it is commonly called, is one likely source of interference because of the fundamental necessity for common types of mixers to be nonlinear. It is true that this possibility is contingent upon prior selectivity being inadequate to rid the mixer of signals so related in frequency as to be capable of producing the mentioned types of interference. This contingency is readily satisfied--indeed, is hard to avoid--when the radio frequencies involved lie in the vhf-uhf portion of the spectrum, because in this region, sharp, highly attenuating absolute selectivity is difficult to realize.

The capability of the mixer to produce significant interference hinges upon two questions. First, is the nonlinearity of the mixer nonlinear element of such a nature that little or no interference will be generated even if the otherwise potentially interfering signals are present? Second, is the circuit configuration in a given case such that the local-oscillator voltage can mix with each of the incoming signals without their mixing appreciably with each other?

The use of infinite series is an expedient and commonly used general approach to problems of interference and amplitude distortion.

Graphs of the transfer characteristics of the nonlinear devices, such as vacuum tubes, used in electronics work are assumed to be power-law functions. A representative generalized series is of the form

$$i = a_o + a_1e + a_2e^2 + a_3e^3 + \dots \tag{3}$$

where i is the current which flows in a given device as a result of applying some voltage e to the device. If e is a sinusoidal voltage, then the series for i expresses some amplification of e, depending upon the impedance through which i is caused to flow; the series also represents the generation of various harmonics of the original pure frequency. On the other hand, if e is the sum of two or more sine waves of different frequencies, then not only does the series show amplification and harmonic distortion, but also it shows the production of new frequencies not initially present and not harmonically related.

Typically, the a's in the series of equation (3), above, get progressively (though not necessarily successively) smaller in magnitude. In analyses involving the series, advantage is taken of this fact in restricting considerations to a limited number of terms. Hence, the series is treated as though it were a finite series. The number of terms retained depends upon how rapidly the coefficients diminish, how large a value of e must be considered, and what orders of magnitude of various components must be taken into account.

Since the a's ordinarily shrink rapidly, the term a_2e^2 is not only the first term giving rise to mixing (through "product demodulation") but is also generally the largest. Hence, a device having a quadratic expansion--i.e., the coefficients of all terms of higher degree than the second in the expansion are zero--could serve as a mixer. A crystal diode approaches this condition, but only imperfectly, whereas a bolometer may be completely disregarded for normal r-f mixer applications because of its very low upper-frequency limit. Laskin[3] refers to:

> ... the assumption that the detector operates within the square law region. This assumption has been taken to be correct for crystals operating at levels below 10 microwatts, and for bolometers over their operating range. However, care must be exercised in each individual setup to insure operation within the square law region.

This same condition regarding crystals is mentioned frequently in the literature by Terman[4] and others. Noise generation and other considerations in a receiver generally preclude a reliance upon small-signal operation of a mixer to the degree mentioned by Laskin in connection with measuring techniques.

Ryder[5] dispels the possibility that a mixer, depending as it does upon some type of nonlinearity for its operation, might generate only negligible interference even if the potentially interfering signals are present. He writes:

> It is apparent that obtaining of the product term of two frequencies is sufficient to bring about demodulation. Since modulation is a process in which a product term is formed, it appears that modulators may also be detectors, and this is true. It has also been demonstrated that a square-law or product type of device yields distortion. In general the square-law device will yield terms representing the sums and differences of all the input frequencies, including the sum and difference of each frequency with itself, thus giving double-frequency and zero-frequency components.

Hence, even a true quadratic device would be subject to nonlinearity-produced interference under certain conditions and would thus rely upon either prior selectivity or the nonexistence of signals at frequencies capable of producing some type of interference within the device. The truth of this is readily demonstrated by substituting

$$e = b_o \sin \omega_o t + b_d \sin \omega_d t + b_a \sin \omega_a t + b_b \sin \omega_b t \tag{4}$$

into the square-law term, $a_2 e^2$, of the power series, where the subscripts denote the following:

subscript o refers to the local oscillator,

subscript d refers to the desired signal and

subscripts a and b refer to two undesired signals.

For this particular example, for the simultaneous application of all four frequencies to the assumed square-law mixer, and for $\omega_{i-f} = \pm (\omega_o - \omega_d)$, then, in addition to the image, i-f interference can occur under one or more of the following conditions:

1. if ω_o, ω_d, ω_a, or $\omega_b = \omega_{i-f}/2$,
2. if $\omega_a = \omega_d$ or $\omega_b = \omega_d$,
3. if $\omega_a = \omega_o$ or $\omega_b = \omega_o$,
4. if $\pm (\omega_a \pm \omega_d)$, $\pm (\omega_a \pm \omega_o)$, $\pm (\omega_b \pm \omega_d)$, or $\pm (\omega_b \pm \omega_o) = \omega_{i-f}$, or
5. if $\pm (\omega_a \pm \omega_b) = \omega_{i-f}$.

Of these, item 2 is a trivial case, and some of the others may be so considered.

However, consider item 5 in the light of the definition for intermodulation which was given in the "Introduction" to this thesis. In conformity with this definition, item 5 presents a case in which the mixer specified output frequency (i-f) is obtained as a direct result of there being applied to the mixer two or more undesired signals such as those at angular frequencies ω_a and ω_b, these frequencies being, in general, different from the desired or the local-oscillator frequencies. It will be recalled that this output is classed as intermodulation when this particular resultant frequency is expressible as the algebraic sum of frequencies which specifically include integral multiples of each of two or more undesired frequencies presented to the mixer input. Consequently, item 5 is classed as intermodulation. More definitely, it is second-order intermodulation.*

A trial by substitution, similar to that used above, soon reveals that the presence, in the power-series expansion of a device, of any term of degree higher than the second introduces multiplied possibilities for intermodulation. It can be shown that when only two undesired signals are present, the number of such possibilities introduced by the n^{th} term is n-1.

It is seen, then, that a distinct and basic incompatibility exists between the elimination of intermodulation (and, similarly, cross-modulation) and the realization of mixing in a device the output

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*It can be demonstrated that the order of the intermodulation can be identified as the sum of the absolute values of the coefficients of the ω's which add algebraically to produce the resultant interfering frequency. Thus, $3\omega_x - \omega_y = \omega_{i-f}$ would represent fourth-order intermodulation existing in a mixer because of interfering frequencies f_x and f_y.

current of which is representable by a power series. Certainly ideal prior selectivity would be a solution which would eliminate the problem on an interchannel basis but would not solve it. Multiple frequencies, such as sidebands, existing within a channel would still interfere with each other in the sense that new frequency components would be produced by them. Moreover, a basic premise of this discussion is that such selectivity is unobtainable.

In consideration of available information regarding present-day active devices such as vacuum tubes, the design theory for a low-interference mixer appears to be most reliably approached from the power-series viewpoint. Yet, as will be apparent later, the complexity of that viewpoint increases rapidly with an increase of degree of the power-series terms. On the other hand, a mixer considered as a power-expansion device is basically dependent upon some higher-degree term--the second, say---being nonzero. From the earlier discussion it is clear that the use of a device having a current characteristic approaching a true quadratic form would exclude a multiplicity of possibilities of interference arising from the series-expansion terms of higher degree than the second. For these reasons, the approach will be taken that at the present state of the art the most practical solution to the problem of obtaining low-interference mixing in a receiver is a device having a current characteristic most nearly approaching a true quadratic function of the applied voltage. In addition it will be assumed that to be satisfactory for general receiver applications, the device must conform well to the quadratic characteristic over a large dynamic range from several volts down into the microvolt region.

The requirement for a large dynamic range at once tends to rule out the use of a crystal diode. Further, the conversion loss in a crystal is considerable. According to Terman,[6] "...only a small fraction of the power supplied by the radio frequency to the crystal appears as intermediate-frequency power in the load. A conversion loss of the order of 8 db is typical." He goes on to point out that (vacuum-tube) diode mixers are more rugged than crystal mixers and have lower conversion loss, frequently giving way, however, to crystal mixers primarily because of the much lower high-frequency limit of a tube as compared to a crystal. He gives the conversion loss for the tube as "commonly of the order of 2 to 4 db" for fundamental operation.

Van der Ziel[7] states that at microwave frequencies a crystal diode mixer gives a smaller noise figure than a vacuum-tube diode mixer. However, he also remarks that at lower frequencies with noise compensation employed, the noise figure for the vacuum-tube diode mixer may be smaller than for a corresponding crystal diode mixer. On the basis of these observations it is reasonable to conclude that an approximate equality of the two cases probably occurs somewhere in the vicinity of the vhf-uhf region.

At the present state of the art, transistors have not advanced to the point that they have been seriously considered as mixers for general vhf-uhf service, although this situation shows promise of changing in the not-too-distant future.

At present it appears that use of a vacuum tube offers one of the most attractive possibilities, at radio frequencies extending downward from a few hundred megacycles, for a receiver mixer having extremely low-intermodulation and cross-modulation capabilities.

Before going on to compare the diode, the triode and screen-grid tubes, it is well to take note of a fact elaborated upon by Slonczewski.[8] While his paper is primarily concerned with pentodes, the fact of immediate interest here is useful for other screen-grid tubes and for triodes. He stresses the point that the region where the graph of transconductance versus grid voltage of a tube is essentially a straight line is the region within which the effects of the higher-degree terms of the plate-current expansion are such that the tube behaves practically as if its current characteristic were a pure quadratic. The following two related observations, of importance to the present study, may be added.

1. With a basis of examination of the characteristics of a large number of receiver tube types, the linear region referred to above establishes class A_1 (finite plate current and zero grid current at all instants of time) as the class of operation in which to work a single-tube mixer if low intermodulation and cross-modulation of higher orders are to be expected.

2. Similarly, within the limits of the linear region mentioned, the assumption that the amplification factor, μ, is a constant is, in general, a good approximation.

With this background, further investigation may now be facilitated by use of equations and series for which validity is dependent upon the conditions of class-A_1 operation and of constant μ.

Now, holding a consideration of multigrid tubes in abeyance, compare the classical equations for the diode and the triode.

For diodes (in space-charge limited continuous conduction)

$$I_{b,D} = K_D \, E_{b,D}^{\alpha} \,, \tag{5}$$

where

$I_{b,D}$ = diode anode space current,

K_D = constant, depending upon the tube type,

$E_{b,D}$ = total instantaneous anode-to-cathode voltage and

α = constant, very nearly 3/2.

For triodes (in class-A_1 operation)

$$I_{b,T} = K_T \left(E_c + \frac{E_{b,T}}{\mu} \right)^{\alpha} \,, \tag{6}$$

where

$I_{b,T}$ = triode anode space current,

K_T = constant, depending upon the tube type,

E_c = total instantaneous grid-to-cathode voltage,

$E_{b,T}$ = total instantaneous anode-to-cathode voltage,

μ = amplification factor (considered constant) and

α = constant, very nearly 3/2.

The Taylor expansions of these two cases are as follows:

for diodes,

$$I_{b,D} = K_D E^{\alpha}_{bo,D} + \alpha K_D E^{\alpha-1}_{bo,D} e_{p,D} + \frac{\alpha(\alpha - 1) K_D E^{\alpha-2}_{bo,D} e^2_{p,D}}{2!} \qquad (7)$$

$$+ \frac{\alpha(\alpha - 1)(\alpha - 2) K_D E^{\alpha-3}_{bo,D} e^3_{p,D}}{3!}$$

$$+ \frac{\alpha(\alpha - 1)(\alpha - 2)(\alpha - 3) K_D E^{\alpha-4}_{bo,D} e^4_{p,D}}{4!} + \ldots , \text{ and}$$

for triodes,

$$I_{b,T} = K_T\left(E_{co} + \frac{E_{bo,T}}{\mu}\right)^{\alpha} + \alpha K_T\left(E_{co} + \frac{E_{bo,T}}{\mu}\right)^{\alpha-1} \cdot \left(e_g + \frac{e_{p,T}}{\mu}\right) \qquad (8)$$

$$+ \frac{\alpha(\alpha - 1) K_T\left(E_{co} + \frac{E_{bo,T}}{\mu}\right)^{\alpha-2}}{2!} \cdot \left(e_g + \frac{e_{p,T}}{\mu}\right)^2$$

$$+ \frac{\alpha(\alpha - 1)(\alpha - 2) K_T\left(E_{co} + \frac{E_{bo,T}}{\mu}\right)^{\alpha-3}}{3!} \cdot \left(e_g + \frac{e_{p,T}}{\mu}\right)^3$$

$$+ \frac{\alpha(\alpha - 1)(\alpha - 2)(\alpha - 3) K_T\left(E_{co} + \frac{E_{bo,T}}{\mu}\right)^{\alpha-4}}{4!} \cdot \left(e_g + \frac{e_{p,T}}{\mu}\right)^4 + \ldots$$

where the addition of an o in a subscript indicates the d-c value at the operating point, and where

$$e_{p,D} = E_{b,D} - E_{bo,D} ,$$
$$e_g = E_c - E_{co} \text{ and}$$
$$e_{p,T} = E_{b,T} - E_{bo,T} .$$

Since the present effort is directed toward finding a condition most nearly approaching quadratic operation, it is of benefit, with each of these series, to examine the ratio of each of the higher-degree terms to the second-degree term. For this purpose let this ratio be called the "relative $n^{\underline{th}}$ degree." For the diode, the relative third degree is

$$\frac{2!\ (\alpha - 2)\ e_{p,D}}{3!\ E_{bo,D}}$$

and the relative fourth degree is

$$\frac{2!\ (\alpha - 2)\ (\alpha - 3)\ e^2_{p,D}}{4!\ E^2_{bo,D}} .$$

For the triode, the relative third degree is

$$\frac{2!\ (\alpha - 2) \left(e_g + \frac{e_{p,T}}{\mu}\right)}{3! \left(E_{co} + \frac{E_{bo,T}}{\mu}\right)}$$

and the relative fourth degree is

$$\frac{2!\ (\alpha - 2)\ (\alpha - 3) \left(e_g + \frac{e_{p,T}}{\mu}\right)^2}{4! \left(E_{co} + \frac{E_{bo,T}}{\mu}\right)^2} .$$

The relative $n^{\underline{th}}$ degree for each case is now readily obtained by induction, if so desired.

For purposes of qualitatively comparing a triode with a diode of about the same size and same normal plate current, it is reasonable to assume, for class-A_1 operation of the triode, that $E_{co} + \frac{E_{bo,T}}{\mu}$ for the triode is of the same order of magnitude as $E_{bo,D}$ for the diode, to a rough approximation. For both a triode and a diode, $\alpha = \frac{3}{2}$, very nearly. Therefore, the ratios

$$\frac{\text{Relative } 3^{\underline{rd}} \text{ degree for the triode}}{\text{Relative } 3^{\underline{rd}} \text{ degree for the diode}} = \frac{e_g + \frac{e_{p,T}}{\mu}}{e_{p,D}}, \tag{9}$$

$$\frac{\text{Relative } 4^{\underline{th}} \text{ degree for the triode}}{\text{Relative } 4^{\underline{th}} \text{ degree for the diode}} = \left(\frac{e_g + \frac{e_{p,T}}{\mu}}{e_{p,D}}\right)^2, \tag{10}$$

and by induction,

$$\frac{\text{Relative } n^{\underline{th}} \text{ degree for the triode}}{\text{Relative } n^{\underline{th}} \text{ degree for the diode}} = \left(\frac{e_g + \frac{e_{p,T}}{\mu}}{e_{p,D}}\right)^{n-2}. \tag{11}$$

Since $e_{p,T}$ and $e_{p,D}$ both are dependent upon plate load impedance, let this impedance be assumed to be zero in order to compare the two classes of tubes directly. Also, let the applied signal and local-oscillator voltages be assumed to be the same for the two classes.

Now, for a triode being operated as a grid mixer, all signals, as well as the local-oscillator voltage, are applied to the grid, so that $e_{p,T} = 0$ under the assumed condition of zero load impedance. For the conducting-diode mixer, $e_{p,D}$ would be the sum of all the applied signals. Hence, the triode grid mixer, operated in class A_1, could be expected to give about the same magnitude of relative n^{th} degree as the conducting-diode mixer.

It is shown later in this paper that for limited-swing class-A_1 operation, the grid-injection plate mixer is equivalent to a grid mixer having only $1/\mu$ as much signal voltage applied but with other considerations equal. As is illustrated by a later development, this equivalent reduction of applied-signal voltage causes undesired interference products resulting from series-expansion terms of higher degree than the second to be reduced at a much faster rate than the desired mixer output arising from the second-degree term. A corollary to this statement is that a class-A_1 plate mixer is capable of handling signals μ times as large as can be handled by the corresponding class-A_1 grid mixer for the same relative-interference production. It must be emphasized that an increasing negligibility of the higher-degree terms of the series expansion is caused by the exponential way in which these terms are reduced by a reduction of applied-signal voltage. Therefore, it follows that for given applied-signal voltages and for values of μ much greater than unity, the plate mixer must represent a much closer approach to a pure quadratic device than do the other types. Hence, the plate mixer should be expected to be the freest of intermodulation and cross-modulation interference relative to the amount of mixer action obtained.

The case of the multigrid tubes may now be dealt with. At high values of plate potential such that the plate current is largely independent of plate voltage, and for class-A_1 operation, the total space current is the sum of the plate and screen currents and is given by the classical form,

$$I_{space} = I_{b,s} + I_{c2} = K_s \left(E_c + \frac{E_{c2}}{\mu_s}\right)^{\alpha} , \tag{12}$$

where

$I_{b,s}$ = anode space current of a multigrid tube,

I_{c2} = screen-grid space current,

K_s = constant depending upon the tube type,

E_c = total instantaneous grid-to-cathode voltage,

E_{c2} = total instantaneous screen-to-cathode voltage,

μ_s = grid-screen transconductance and

α = constant, very nearly 3/2.

It is seen that the equation for the space current of a screen-grid tube is of the same form as the equation for the plate current of a triode. There is certainly no question but that an ordinary screen-grid tube, if triode connected, will operate in all respects like a triode. However, if the triode connection is not used, then the utilization of the space-current characteristic can hardly be predicted in a generalized way because of the dependence upon the many circuit configurations which might be devised. Moreover, screen-grid tubes are, in general, much noisier than triodes and are not as well suited for vhf-uhf work as are comparable triodes. Therefore, screen-grid tubes per se are not considered for the remainder of this study.

Some interesting possibilities for mixers are offered by the suggestion of the use of feedback. However, practical considerations, in general, rule out the advisability of giving serious consideration in this thesis to the idea of using feedback at vhf-uhf frequencies for the purpose of reducing intermodulation and cross-modulation interference. Positive feedback is difficult to control at low frequencies and next to impossible at 200 to 400 mc. In this frequency region it is hard to accomplish negative feedback and harder to keep it from turning to positive feedback at some unexpected frequency.

Apart from these difficulties, there appears to be a completely feasible possibility that negative feedback could be used to accomplish a great deal of intermodulation and cross-modulation reduction. Again consider the triode plate-current series expansion, which has a counterpart in the space-current expansion for multigrid tubes. This has been shown to have the form

$$I_b = \sum_{n=0}^{\infty} A_n \left(e_g + \frac{e_p}{\mu}\right)^n . \tag{13}$$

The same binomial, $e_g + \frac{e_p}{\mu}$, gives rise to mixing, intermodulation and cross-modulation, all three, by virtue of being raised to a power, n. Since each term of the series, including the term corresponding to mixing, is thus subject to the invariable binomial expansion, it appears that any prospect for feedback reduction of interference lies in dealing with the binomial.

Therefore, with voltage feedback assumed, let the binomial be separated into its component parts. For the grid mixer the result can be represented as

$$e_g + \frac{e_p}{\mu} = e_{osc.} + e_{desired} + e_{undesired} + e_{feedback} \tag{14}$$
$$- \frac{1}{\mu}\, e\left[i_p(f),\ Z(f)\right],$$

where the symbolism of the last term on the right-hand side signifies a voltage function of plate current and plate-load impedance, each of the latter two being, in turn, a function of frequency. For the grid-injection plate mixer the corresponding result is

$$e_g + \frac{e_p}{\mu} = e_{osc.} + \frac{1}{\mu}\left(e_{desired} + e_{undesired} + e_{feedback}\right) \tag{15}$$
$$- \frac{1}{\mu}\, e\left[i_p(f),\ Z(f)\right].$$

Ideally, in either case if it were possible to make

$$e_{feedback} = - e_{undesired}\ , \tag{16}$$

then all interference could be eliminated. However, this ideal would be impossible to realize for two reasons.

1. Since $e_{feedback}$ must be some function of the output current, the output current must include some function of $e_{undesired}$ if the feedback is to be effective. This requirement would directly contradict the hypothesized ideal case.

2. In order for $e_{feedback}$ not to include some feedback of $e_{desired}$, the feedback circuit would have to involve an ideal selective circuit capable of rejecting just the desired channel and nothing else. Such a selective circuit is not physically realizable.

However, the way is pointed for obtaining feedback reduction of interference.

Consideration of equation (16) indicates that the feedback should be at the same frequencies as the undesired signals and should be negative. Since the undesired signals might lie at any frequency, the feedback network, whether passive or active, should be of the band-elimination type, passing all frequency components equally with the exception of a narrow band one channel-width wide and centered about the desired frequency. As mentioned in the last paragraph, a condition of zero feedback within this band would be desirable.

As a corollary to the foregoing observations regarding r-f feedback, it would seem likely that a mixer (for some specialized purpose) employing negative feedback at the desired frequency and nowhere else would suffer a consequent relative degradation with regard to intermodulation and cross-modulation interference. The "Summary" of an interesting article by Tucker[9] indicates that a reduction of nonlinear distortion in a frequency changer can be realized by use of a feedback scheme which he sets forth, an arrangement employing a second similar frequency changer in the feedback path. In the article itself, however, it is not obvious that the claim for reduction of nonlinear distortion applies to frequency changers.

The idea of obtaining an improvement of <u>r-f</u> intermodulation and cross-modulation interference generation in a mixer by means of feeding back the resultant i-f does not seem promising. However, there probably would be a relative reduction in <u>i-f</u> intermodulation and cross-modulation products, i.e., the products formed within the i-f pass band by signals of other frequencies also within this band. The advantage gained by this scheme would be partially or totally offset by the fact that there would be new intermodulation products formed between multiples of the intermediate frequencies and the incoming r-f signal frequencies. Also, in vhf-uhf applications, the intermediate frequencies (for image reasons) are generally high enough so that again considerable difficulty would, in all probability, be experienced in attempting to obtain controlled feedback. Boggs[10,11] has investigated mixers employing both voltage and current feedback at the intermediate frequency, but in the two articles to which reference is made here, there appear to be no claims for a reduction of nonlinear distortion.

CHAPTER III

DESIGN CONSIDERATIONS FOR A LOW-INTERMODULATION TRIODE PLATE MIXER

The triode plate mixer has been shown, in a general way, to offer a low-intermodulation (and cross-modulation) means of obtaining mixing action. The present chapter undertakes to determine conditions of operation to make use of these characteristics.

The desirability of class-A_1 operation of the low-intermodulation mixer has been brought out previously. In addition, the limits of the region of lowest production of higher-order interference components have been approximated[12] as those established by the linear portion of a g_m-vs-E_c characteristic for a tube. Because each tube type (only triodes being considered) has a unique g_m-vs-E_c curve corresponding to each static transfer characteristic and, consequently, to each plate voltage, a large range of possibilities exists from which a selection of operating point must be made. It might be surmised that the center point of the linear section of the chosen g_m-vs-E_c characteristic would be a logical operating point. Even if this were true, the end points of this linear region would normally be so indefinite that the center could not be accurately found by ordinary bisection.

A less brief but a much more precise procedure for choosing an operating point appears as a result of the following development. Also, criteria for the circuit parameters are determined.

The first step in deciding upon an operating point is to pick a best g_m-vs-E_c characteristic for a given tube type. Obviously, a family of these characteristics for the tube is required. The following three criteria, taken collectively, will serve as a guide. The particular characteristic to be selected should

1. most nearly approach a true straight line over an appreciable grid-voltage range,

2. permit the largest total grid swing within the linear portion, and

3. have the steepest slope of the linear segment.

The first of these criteria has already been discussed. The reasons for the second and third will appear in subsequent development. With three criteria to be satisfied, the choice of a particular characteristic is somewhat a matter of judgement if an inordinate amount of time is not to be spent in finding a unique theoretically best point. As will become apparent, the selection of a characteristic usually need not be highly critical.

Until now, nothing has been said regarding the choice of a tube type. If the three items just listed are a legitimate guide in the selection of the best g_m-vs-E_c characteristic for a given tube, it follows that they may serve equally well, in precisely the same manner, in the comparison of various types the characteristics of which have all been plotted to the same scales or for which the equivalent information has been tabulated.

Now, on the presumption that a tube type and a particular g_m-vs-E_c characteristic have been selected, consider the Taylor series given by

Terman[13] to describe the plate-current characteristic of a grid-controlled vacuum tube. Valid for the limited-swing class-A_1 operation with constant μ being assumed here, it is

$$i_p = g_m\left(e_g + \frac{e_p}{\mu}\right) + \frac{1}{2!}\frac{\partial g_m}{\partial E_c}\left(e_g + \frac{e_p}{\mu}\right)^2 + \frac{1}{3!}\frac{\partial^2 g_m}{\partial E_c{}^2}\left(e_g + \frac{e_p}{\mu}\right)^3 + \ldots. \tag{17}$$

Assume that the plate circuit of the tube so described contains only a passive load resistance R(f), or simply R, a function of frequency. It can then be agreed (for the case of a triode) that the plate voltage, e_p, depends upon R(f) and also upon the plate current $i_p(f)$, or simply i_p, which is a function both of frequency and of the voltage, e_g, applied between grid and cathode. Hence, the series as shown by equation (17) can be viewed as an implicit equation for i_p.

However, the series in inverted form is desired so that i_p is presented as an explicit function of e_g in the form[14]

$$i_p = \sum a_1 e_g + \sum a_2 e_g{}^2 + \sum a_3 e_g{}^3 + \sum a_4 e_g{}^4 + \ldots , \tag{18}$$

where the summation signs are in anticipation of a different value of coefficient for each frequency arising in each term, the load resistance having been previously stated to be a function of frequency and the coefficients being functions of load resistance and hence of frequency, as will be seen. The intended significance of (18) is actually more precisely expressed by the form

$$i_p = \sum_j \sum_i a_{ij} e_g^{\,j}.$$

By a method based on a technique rigorously developed by Carson, [15] the inverted series can be written in a symbolism which is basically that given by Terman.[16] It is

$$i_p = \frac{\mu e_g}{r_p + R_1} + \frac{1}{2!\mu g_m}\frac{\partial g_m}{\partial E_c}\frac{e_1^{\,2}}{r_p + R_2} \tag{19}$$

$$+ \frac{\dfrac{1}{3!\mu^2 g_m}\dfrac{\partial^2 g_m}{\partial E_c^{\,2}} e_1^{\,3} - \dfrac{1}{\mu g_m}\dfrac{\partial g_m}{\partial E_c} e_1 e_2}{r_p + R_3}$$

$$+ \frac{\dfrac{1}{4!\mu^3 g_m}\dfrac{\partial^3 g_m}{\partial E_c^{\,3}} e_1^{\,4} - \dfrac{3}{3!\mu^2 g_m}\dfrac{\partial^2 g_m}{\partial E_c^{\,2}} e_1^{\,2} e_2 - \dfrac{1}{2!\mu g_m}\dfrac{\partial g_m}{\partial E_c}\left(2e_1 e_3 - e_2^{\,2}\right)}{r_p + R_4} \cdots,$$

where the notation, the same as that employed by Terman, is as follows:

i_p = total instantaneous plate-current component which varies about the operating-point value,

e_g = total variable component of the instantaneous grid voltage,

μ = operating-point value of amplification factor, assumed constant,

r_p = operating-point value of dynamic plate resistance, assumed constant,

g_m = operating-point value of transconductance, with the successive partials of g_m also taken at the operating point,

R_n = load resistance offered to the $n^{\underline{th}}$ -order component of plate current, this resistance corresponding to $R(f)$, a function of frequency, and requiring the substitution of the appropriate resistance value for each frequency term of the $n^{\underline{th}}$ -order component,

$e_1 = \mu e_g \dfrac{r_p}{(r_p + R_1)}$ = voltage drop produced in plate resistance by first-order component of plate current,

$e_2 = \dfrac{R_2}{(r_p + R_2)} \dfrac{1}{2!\mu g_m} \dfrac{\partial g_m}{\partial E_c} e_1{}^2$ = voltage drop produced across the load resistance by the second-order component of plate current and

$$e_3 = \frac{R_3}{(r_p + R_3)}\left(\frac{1}{3!\mu^2 g_m} \frac{\partial^2 g_m}{\partial E_c{}^2} e_1{}^3 - \frac{1}{\mu g_m} \frac{\partial g_m}{\partial E_c} e_1 e_2\right)$$

= voltage drop produced across the load resistance by the third-order component of plate current.

A brief study of (19) and its associated definitions reveals that it does conform to equation (18) if (18) is rewritten in the form

$$i_p = \sum i_1 + \sum i_2 + \sum i_3 + \ldots + \sum i_n + \ldots , \tag{20}$$

where, from (18) and (19),

$\sum i_1 = \sum a_1 e_g = \dfrac{\mu}{(r_p + R_1)} e_g$ = first-order components of plate current,

$$\sum i_2 = \sum a_2 e_g{}^2 = \frac{1}{2!\mu g_m} \frac{\partial g_m}{\partial E_c} \frac{1}{(r_p + R_2)} e_1{}^2$$

(e_1 being a function of e_g as well as of load resistance)

= second-order components of plate current,

$\sum i_3 = \sum a_3 e_g^{\ 3} = \ldots$, etc.

The expressions above are correct only when proper interpretation is given to the effects of R_n and e_n in the right-hand sides. Consequently, the natures of R_n and e_n and their proper employment in (19) must be stressed. The load resistance R_n is a function of frequency, and for each frequency component occurring within a given order of plate current, the value of the resistance peculiar to that frequency must be employed. If a frequency component arises in more than one order, the same value of resistance will hold at that frequency in all the orders in which the particular frequency component occurs. Thus, in connection with $\sum i_1$, suppose that e_g is the sum of three voltages, each at a different frequency, as follows:

$$e_g = e_o + e_a + e_b = E_o \cos \omega_o t + E_a \cos \omega_a t + E_b \cos \omega_b t \ , \tag{21}$$

where

$$e_o = E_o \cos \omega_o t \ ,$$
$$e_a = E_a \cos \omega_a t \text{ and}$$
$$e_b = E_b \cos \omega_b t.$$

Then,

$$\sum i_1 = \frac{\mu e_g}{(r_p + R_1)} = \frac{\mu}{(r_p + R_o)} E_o \cos \omega_o t + \frac{\mu}{(r_p + R_a)} E_a \cos \omega_a t \tag{22}$$
$$+ \frac{\mu}{(r_p + R_b)} E_b \cos \omega_b t \ ,$$

where R_o, R_a and R_b are the values of R_1 at frequencies ω_o, ω_a and ω_b, respectively. Then, in keeping with the definition given for e_1, i.e.,

$$e_1 = \mu e_g \frac{r_p}{(r_p + R_1)} = \text{voltage drop produced in plate resistance by the first-order component of plate current,}$$

it is seen that

$$e_1 = r_p \sum i_1 = \sum i_1 r_p \tag{23}$$

$$= \frac{\mu r_p}{(r_p + R_o)} E_o \cos \omega_o t + \frac{\mu r_p}{(r_p + R_a)} E_a \cos \omega_a t$$

$$+ \frac{\mu r_p}{(r_p + R_b)} E_b \cos \omega_b t.$$

By use of equation (23) for e_1, the nature of $\sum i_2$ may be understood more readily, since it has already been given that

$$\sum i_2 = \frac{1}{2! \mu g_m} \frac{\partial g_m}{\partial E_c} \frac{1}{(r_p + R_2)} e_1^2 .$$

It can be seen that the squaring of e_1 will give rise to current components in $\sum i_2$ which are at the double and the sum and the difference frequencies for the frequencies initially present in e_g. These components are dependent in amplitude both upon the original amplitudes of the components of e_g and upon the value of the plate load resistance at each of the original frequencies. In addition, it should be emphasized that R_2 may have a separate value at each frequency produced by squaring e_1, in a fashion completely analagous to the several values assumed by R_1 at the frequencies of the components of e_g. If the frequencies resulting from squaring e_1 are 0 (d.c.), $2\omega_o$, $2\omega_a$, $2\omega_b$, $\omega_a - \omega_o$, $\omega_a + \omega_o$, $\omega_b - \omega_o$, $\omega_b + \omega_o$, $\omega_a - \omega_b$ and $\omega_a + \omega_b$, then

$$\sum i_2 = i_{d.c.} + i_{2\omega_o} + i_{2\omega_a} + i_{2\omega_b} + i_{\omega_a-\omega_o} + i_{\omega_a+\omega_o} + i_{\omega_b-\omega_o} + i_{\omega_b-\omega_o} + i_{\omega_a-\omega_b} + i_{\omega_a-\omega_b} \tag{24}$$

and

$$e_2 = \sum i_2 R_2 = i_{d.c.}R_{d.c.} + i_{2\omega_o}R_{2\omega_o} + i_{2\omega_a}R_{2\omega_a} + i_{2\omega_b}R_{2\omega_b} + i_{\omega_a-\omega_b}R_{\omega_a-\omega_b} + i_{\omega_a+\omega_o}R_{\omega_a+\omega_o} + i_{\omega_b-\omega_o}R_{\omega_b-\omega_o} + i_{\omega_b+\omega_o}R_{\omega_b+\omega_o} + i_{\omega_a-\omega_b}R_{\omega_a-\omega_b} + i_{\omega_a+\omega_b}R_{\omega_a+\omega_b}. \tag{25}$$

Now, for the purpose of gaining insight, suppose that R_2 were zero at all frequencies contained in i_2 except at $\omega_a-\omega_o$. Then e_2 would have but one component, $i_{\omega_a-\omega_o}R_{\omega_a-\omega_o}$. Hence, the single-frequency value of $R_2 = R_{\omega_a-\omega_o}$ would not only affect the values of $\sum i_2$ at all frequencies 0, $2\omega_o$, $2\omega_a$, $2\omega_b$, $\omega_a-\omega_o$, etc., and would limit e_2 to a single-frequency component, but would also affect higher-order currents and voltages dependent upon e_2 as shown in (19).

In a fashion which is but an extension of that just demonstrated, all the higher-order currents and equivalent voltages are functions of all the lower-order currents and associated values assumed by the plate load resistance as a function of frequency. Hence, i_1 is a function of e_g, $e_1 = \sum i_1 r_p$ is a function of e_g, i_2 is a function of e_1^2 and hence of e_g^2, e_2 is a function of i_2 and hence of e_g^2, i_3 is a function of e_1^3 and of $e_1 e_2$ and hence of e_g^3 in a fairly complicated way, etc.

The intention of the foregoing interpretation of the analysis is to provide as clear a view as practical of the mechanism by which synthetic frequency components, and hence interference, are generated within a vacuum tube. Certainly a review of the foregoing makes clear that all possible frequencies corresponding to the $n^{\underline{th}}$ order can be found by raising e_g to the $n^{\underline{th}}$ power. It is of interest now to employ this device to find the possible frequencies arising out of, say, the first four orders of plate current, it being reasonable to expect, on the basis of experience, that orders higher than about the fourth will usually be of rapidly diminishing relative importance. For this determination, e_g will be assumed to be the sum of three voltages of different frequencies. This assumption is made for simplicity, three frequencies--the local-oscillator frequency and two signals--ordinarily being the minimum number which will produce intermodulation in a mixer. Thus, again let

$$e_g = e_o + e_a + e_b = E_o \cos \omega_o t + E_a \cos \omega_a t + E_b \cos \omega_b t. \qquad (26)$$

Then,

$$e_g^2 = e_o^2 + e_a^2 + e_b^2 + 2e_o e_a + 2e_o e_b + 2e_a e_b \qquad (27)$$

$$= \frac{E_o^{\,2} + E_a^{\,2} + E_b^{\,2}}{2} + \frac{E_o^{\,2}}{2} \cos 2\omega_o t + \frac{E_a^{\,2}}{2} \cos 2\omega_a t + \frac{E_b^{\,2}}{2} \cos 2\omega_b t$$

$$+ E_o E_a \cos(\omega_a - \omega_o)t + E_o E_a \cos(\omega_a + \omega_o)t$$

$$+ E_o E_b \cos(\omega_b - \omega_o)t + E_o E_b \cos(\omega_b + \omega_o)t$$

$$+ E_a E_b \cos(\omega_a - \omega_b)t + E_a E_b \cos(\omega_a + \omega_b)t.$$

Next,

$$e_g^{\,3} = e_o^{\,3} + e_a^{\,3} + e_b^{\,3} + 3e_o^{\,2}e_a + 3e_o^{\,2}e_b + 3e_a^{\,2}e_o + 3e_b^{\,2}e_o \tag{28}$$

$$+ 3e_a^{\,2}e_b + 3e_b^{\,2}e_a + 6e_o e_a e_b$$

$$= \frac{3(E_o^{\,3} + 2E_a^{\,2}E_o + 2E_b^{\,2}E_o)}{4} \cos \omega_o t$$

$$+ \frac{3(E_a^{\,3} + 2E_o^{\,2}E_a + 2E_b^{\,2}E_a)}{4} \cos \omega_a t$$

$$+ \frac{3(E_b^{\,3} + 2E_o^{\,2}E_b + 3E_a^{\,2}E_b)}{4} \cos \omega_b t$$

$$+ \frac{E_o^{\,3}}{4} \cos 3\omega_o t + \frac{E_a^{\,3}}{4} \cos 3\omega_a t + \frac{E_b^{\,3}}{4} \cos 3\omega_b t$$

$$+ \frac{3E_o^{\,2}E_a}{4} \left[\cos(2\omega_o - \omega_a)t + \cos\,(2\omega_o + \omega_a)t\right]$$

$$+ \frac{3E_o^{\,2}E_b}{4} \left[\cos(2\omega_o - \omega_b)t + \cos\,(2\omega_o + \omega_b)t\right]$$

$$+ \frac{3E_a^{\,2}E_o}{4}\left[\cos(2\omega_a - \omega_o)t + \cos(2\omega_a + \omega_o)t\right]$$

$$+ \frac{3E_b^{\,2}E_o}{4}\left[\cos(2\omega_b - \omega_o)t + \cos(2\omega_b + \omega_o)t\right]$$

$$+ \frac{3E_a^{\,2}E_b}{4}\left[\cos(2\omega_a - \omega_b)t + \cos(2\omega_a + \omega_b)t\right]$$

$$+ \frac{3E_b^{\,2}E_a}{4}\left[\cos(2\omega_b - \omega_a)t + \cos(2\omega_b + \omega_a)t\right]$$

$$+ \frac{3E_oE_aE_b}{2}\left[\cos(\omega_o + \omega_a - \omega_b)t + \cos(\omega_o + \omega_b - \omega_a)t\right.$$

$$\left.+ \cos(\omega_a + \omega_b - \omega_o)t + \cos(\omega_a + \omega_b + \omega_o)t\right].$$

Finally,

$$\begin{aligned} e_g^{\,4} &= e_o^{\,4} + e_a^{\,4} + e_b^{\,4} + 4e_o^{\,3}e_a + 4e_o^{\,3}e_b + 4e_a^{\,3}e_o \qquad (29) \\ &+ 4e_b^{\,3}e_o + 4e_a^{\,3}e_b + 4e_b^{\,3}e_a \\ &+ 6e_o^{\,2}e_a^{\,2} + 6e_o^{\,2}e_b^{\,2} + 6e_a^{\,2}e_b^{\,2} \\ &+ 12e_o^{\,2}e_ae_b + 12e_a^{\,2}e_oe_b + 12e_b^{\,2}e_oe_a \\ &= \frac{3(E_o^{\,4} + E_a^{\,4} + E_b^{\,4} + 4E_o^{\,2}E_a^{\,2} + 4E_o^{\,2}E_b^{\,2} + 4E_a^{\,2}E_b^{\,2})}{8} \\ &+ \frac{E_o^{\,4} + 3E_o^{\,2}E_a^{\,2} + 3E_o^{\,2}E_b^{\,2}}{2}\cos 2\omega_o t \end{aligned}$$

$$+ \frac{E_a^{\,4} + 3E_a^{\,2}E_o^{\,2} + 3E_a^{\,2}E_b^{\,2}}{2} \cos 2\omega_a t$$

$$+ \frac{E_b^{\,4} + 3E_b^{\,2}E_o^{\,2} + 3E_b^{\,2}E_a^{\,2}}{2} \cos 2\omega_b t$$

$$+ \frac{E_o^{\,4}}{8} \cos 4\omega_o t + \frac{E_a^{\,4}}{8} \cos 4\omega_a t + \frac{E_b^{\,4}}{8} \cos 4\omega_b t$$

$$+ \frac{3(E_o^{\,3}E_a + E_a^{\,3}E_o + 2E_b^{\,2}E_oE_a)}{2} \left[\cos(\omega_a - \omega_o)t + \cos(\omega_a + \omega_o)t\right]$$

$$+ \frac{3(E_o^{\,3}E_b + E_b^{\,3}E_o + 2E_a^{\,2}E_oE_b)}{2} \left[\cos(\omega_b - \omega_o)t + \cos(\omega_b + \omega_o)t\right]$$

$$+ \frac{3(E_a^{\,3}E_b + E_b^{\,3}E_a + 2E_o^{\,2}E_aE_b)}{2} \left[\cos(\omega_a - \omega_b)t + \cos(\omega_a + \omega_b)t\right]$$

$$+ \frac{3E_o^{\,2}E_a^{\,2}}{4} \left[\cos 2(\omega_a - \omega_o)t + \cos 2(\omega_a + \omega_o)t\right]$$

$$+ \frac{3E_o^{\,2}E_b^{\,2}}{4} \left[\cos 2(\omega_b - \omega_o)t + \cos 2(\omega_b + \omega_o)t\right]$$

$$+ \frac{3E_a^{\,2}E_b^{\,2}}{4} \left[\cos 2(\omega_a - \omega_b)t + \cos 2(\omega_a + \omega_b)t\right]$$

$$+ \frac{E_a^{\,3}E_o}{2} \left[\cos (3\omega_a - \omega_o)t + \cos (3\omega_a + \omega_o)t\right]$$

$$+ \frac{E_b^{\,3}E_o}{2} \left[\cos (3\omega_b - \omega_o)t + \cos (3\omega_b + \omega_o)t\right]$$

$$+ \frac{E_a{}^3 E_b}{2} \left[\cos\,(3\omega_a - \omega_b)t + \cos\,(3\omega_a + \omega_b)t\right]$$

$$+ \frac{E_b{}^3 E_a}{2} \left[\cos\,(3\omega_b - \omega_a)t + \cos\,(3\omega_b + \omega_a)t\right]$$

$$+ \frac{E_o{}^3 E_a}{2} \left[\cos\,(3\omega_o - \omega_a)t + \cos\,(3\omega_o + \omega_a)t\right]$$

$$+ \frac{E_o{}^3 E_b}{2} \left[\cos\,(3\omega_o - \omega_b)t + \cos\,(3\omega_o + \omega_b)t\right]$$

$$+ \frac{3E_o{}^2 E_a E_b}{2} \left[\cos\,(2\omega_o + \omega_a - \omega_b)t + \cos\,(2\omega_o + \omega_b - \omega_a)t\right.$$

$$\left. + \cos\,(\omega_a + \omega_b - 2\omega_o)t + \cos\,(\omega_a + \omega_b + 2\omega_o)t\right]$$

$$+ \frac{3E_a{}^2 E_o E_b}{2} \left[\cos\,(2\omega_a + \omega_o - \omega_b)t + \cos\,(2\omega_a + \omega_b - \omega_o)t\right.$$

$$\left. + \cos\,(\omega_o + \omega_b - 2\omega_a)t + \cos\,(\omega_o + \omega_b + 2\omega_a)t\right]$$

$$+ \frac{3E_b{}^2 E_o E_a}{2} \left[\cos\,(2\omega_b + \omega_o - \omega_a)t + \cos\,(2\omega_b + \omega_a - \omega_o)t\right.$$

$$\left. + \cos\,(\omega_o + \omega_a - 2\omega_b)t + \cos(\omega_o + \omega_a + 2\omega_b)t\right].$$

Since the present discussion is concerned with the vacuum tube used as a mixer, it is realistic to hypothesize that the range of input-signal frequencies which must be dealt with here can be greatly restricted by prior selectivity, i.e., by r-f (radio-frequency) selectivity inserted in passive and/or active circuits between the signal source and the mixer. Thus, aside from the local-oscillator voltage, the only signals which will be presented to the mixer input will be those quite near in frequency to the frequency of the signal which is desired. At the same time, it must be recognized that no known means exists for attaining ideal selectivity whereby all signals on channels other than the desired channel can be eliminated without a considerable, or at best an appreciable, waste of spectrum space being incurred by the excessively large channel-to-channel frequency spacing required.

It is also practical, in connection with the mixer, to think in terms of having considerable subsequent selectivity such that the only mixer-output components which will be passed by i-f (intermediate-frequency) amplifiers following the mixer will be those within a very narrow spectral region centered about the i-f carrier frequency for which the equipment has been designed. As is almost universally the case, the intermediate frequency will here be assumed to be much lower than either the local-oscillator frequency or the desired radio frequencies, although still considerably above the highest audio or video frequency with which the incoming signals may be modulated.

Another assumption which will be used here in making a preliminary examination of the frequencies appearing in equations (26) through (29) is that the local-oscillator frequency lies below the radio frequency to which the equipment is tuned. To have the local-oscillator frequency above the desired-signal frequency would serve as well, but it is expedient here to settle upon one or the other.

Remembering now that equations (26) through (29) show all possible frequencies which may be generated by the first- through the fourth-order terms in the plate-current expansion when two incoming carriers and a local-oscillator voltage are applied to the mixer, let the subscript o denote the local oscillator and the subscripts a and b denote the two incoming signals. Upon application of the assumptions just made, it is immediately evident that all mixer output frequencies which are integral multiples of the initial frequencies may be discarded by virtue of the rejection afforded by the i-f selectivity. All of the first-order terms shown by equation (26) fall within this category, since each frequency shown is an initial frequency (including the local-oscillator frequency) multiplied by unity.

Next, all output frequencies which are arithmetic sums of initial frequencies and/or their multiples, with all signs alike in any one component, will obviously lie well above the intermediate frequency and will be rejected by the i-f selectivity. On the other hand, "constant" or "d-c" terms and frequencies which are the simple differences of the two incoming frequencies (the two necessarily being very nearly equal because of r-f selectivity) will be rejected by the i-f selectivity because they will lie well below the intermediate frequency. All but two of the second-order terms shown by equation (27) have been eliminated at this point. These two show simple second-order mixing of the local-oscillator frequency with each of the incoming-signal frequencies. Mixer outputs corresponding to either of these terms might be passed by the i-f circuits according to such assumptions as have so far been made. The two remaining second-order components are

$$E_o E_a \cos(\omega_a - \omega_o)t \tag{30}$$

and

$$E_oE_b \cos(\omega_b - \omega_o)t. \tag{31}$$

With the understanding that

$$\omega_{i\text{-}f} = \omega_{desired} - \omega_o \tag{32}$$

and that ω_a and ω_b are both approximately equal to $\omega_{desired}$, it can be readily seen that every term having its frequency given in the form

$$2\omega_o - \omega_x \quad \text{or} \quad 2\omega_x - \omega_o \tag{33}$$

will be rejected by the i-f selectivity because the frequency will be displaced from the intermediate frequency by approximately the local-oscillator frequency in the first case and by approximately the signal frequency in the second. Also, the frequencies

$$2\omega_a - \omega_b \quad \text{and} \quad 2\omega_b - \omega_a \tag{34}$$

will each be in the r-f region and so will be removed by i-f rejection. The same thing holds for the remaining three third-order terms of equation (28) which have not already been eliminated, these three terms being at the frequencies

$$\omega_o + \omega_a - \omega_b, \tag{35}$$

$$\omega_o + \omega_b - \omega_a \tag{36}$$

and

$$\omega_a + \omega_b - \omega_o. \tag{37}$$

It is now clear that for the two-signal case, at least, the only components, up through the third order, which are not eliminated by quite general constraints are those second-order components which give rise to mixing. Closer examination reveals that the foregoing statement is also true where more than two signals are involved. In particular, it is interesting to note that for the mixer, outputs resulting from the first two odd orders can be entirely removed by ordinary design procedure. In fact, extension of the trial process used above shows that in general, for the case of a mixer, the first several of the odd-order outputs can be eliminated. However, the same cannot be said for the even-order currents, even of the lowest orders, although many components of these need not be bothersome, while the second-order current provides useful output, as was pointed out previously. It will presently be seen that there are undesired components of the fourth-order current [see equation (29)] which ordinary selectivity cannot eliminate. This is true because once two or more signals bearing certain frequency relationships to each other get past the imperfect r-f selectivity to the mixer input, and if the mixer tube characteristic has a nonzero fourth-degree term in its power-series expansion [see equations (18) and (19)], intermodulation resultants can appear either at the intermediate frequency or sufficiently nearby to fall well within the i-f pass band where no ordinary filtering techniques can thenceforth remove them.

Many of the fourth-degree components need no further mention because they fall within one of the categories already discussed. It will be noticed, however, that the frequencies

$$\omega_a - \omega_o \quad \text{and} \quad \omega_b - \omega_o \tag{38}$$

reappear, these frequencies having appeared before in the second-degree term and having been noted therewith to be those which account for ordinary mixing. The fact that these same frequencies arise in the second- and fourth-degree terms--and indeed they can be shown to arise in all subsequent even-degree terms--warrants a brief digression here. It can be observed that every higher even power of e_g will produce all the frequencies produced by every smaller even power. Similarly, every higher odd power will produce all the frequencies produced by every smaller odd power.

In view of these observations and in connection with what has been said earlier regarding "order," it can now be pointed out that in the lowest-degree term to produce a frequency of a given order, the degree and the order are numerically the same. This being the case, an effort to reduce or eliminate interference generation by the method of reducing or eliminating the production of a given order must concern itself primarily with the lowest degree term producing that order. For this to be accepted, reliance must be placed upon the comparative negligibility of the magnitudes of all higher degree terms capable of producing interfering frequencies. This reliance, however, must be reinforced by a judicious limiting of the maximum amplitude of any signal voltage permitted to reach the mixer.

In light of the foregoing digression, the components of the fourth-degree term which have not already been shown to be eliminated will now be examined critically.

The second-order components previously noted as resulting from the fourth-degree term are

$$\frac{3(E_o{}^3E_a + E_a{}^3E_o + 2E_b{}^2E_oE_a)}{2} \cos(\omega_a - \omega_o)t \tag{39}$$

and

$$\frac{3(E_o{}^3E_b + E_b{}^3E_o + 2E_a{}^2E_oE_b)}{2} \cos(\omega_b - \omega_o)t. \tag{40}$$

Compare these with the second-order components previously shown for the second-degree term, namely,

$$E_oE_a \cos(\omega_a - \omega_o)t \tag{41}$$

and

$$E_oE_b \cos(\omega_b - \omega_o)t. \tag{42}$$

Suppose, for the moment, that $E_a \cos \omega_a t$ were the original desired r-f signal and that $E_b \cos \omega_b t$ were an undesired signal in some frequency channel different from the desired channel but near enough in frequency to be passed to the mixer input by the r-f selective circuits. The separation between $\omega_a - \omega_o$ and $\omega_b - \omega_o$ would be identical, on an absolute frequency basis, to that between ω_a and ω_b. If E_a and E_b, instead of being constant amplitudes, should include amplitude-modulation functions (similar in form but, in general, not the same) such that each could be represented in the form

$$E_x = E_{max}(1 + m \cos \omega_m t), \tag{43}$$

then both the desired and the undesired r-f signals would occupy frequency bands of nonzero width, there being one sideband frequency above and one below the carrier for each modulating frequency in each of the two cases. With second-degree mixing, the respective bandwidths would be the same at the converted (intermediate) frequency as at the original (radio) frequency for each case. Suppose that the maximum modulating frequency ever to be employed is $\omega_{m,max}$, the same for both the desired and the undesired channels. For double-sideband amplitude modulation, an i-f bandwidth barely in excess of $2\omega_{m,max}$ would suffice to pass all the required intelligence contained in the converted desired signal, if the converted signal were centered in the i-f pass band. For second-degree mixing, even if the modulating frequency spectrum were continuous from zero to $\omega_{m,max}$, a channel-to-channel frequency spacing, on centers, slightly greater than $2\omega_{m,max}$ would be sufficient to permit the i-f selectivity to reject the entire undesired channel.* Thus, no term lower than the fourth degree in the plate-current series expansion should ordinarily cause any difficulty.

However, the second-order component of the fourth-degree term presents an undesirable situation, even if the undesired signal, $E_b \cos \omega_b t$, should vanish. In this case E_b would equal zero, and the component which would unavoidably be passed by the i-f response (if any reception at all were to be realized) would be

$$\frac{3(E_o{}^3 E_a + E_a{}^3 E_o)}{2} \cos(\omega_a - \omega_o)t. \tag{44}$$

*Such factors as i-f skirt selectivity, frequency drift, and Doppler effects would require somewhat greater channel spacing and i-f bandwidth but are being neglected here.

With E_o assumed constant but with E_a having the form shown by equation (43), a glance at a table of trigonometric identities is sufficient to disclose that the presence of E_a^3 in the coefficient of (44) will produce distortion. The relative severity of this distortion will depend upon the relative magnitudes of E_o and E_a and will be minimized if $E_o \gg E_a$, the distortionless-mixing portion of the coefficient having E_o^3 as a factor. If, in addition to the desired signal, the undesired signal, also modulated, is present (i.e., E_b is nonzero), the second-order component of the fourth-degree term is given by (39), from which it can be seen that both distortion and cross-modulation interference will result. Again, the greater E_o can be made, relative to E_b as well as E_a, the further the relative distortion and cross-modulation effects being presently examined may be reduced. On the other hand, the second-degree, second-order desired output of the mixer shown by (30) is enhanced in direct proportion to an increase in E_o (within the limits of class-A_1 operation).

In light of these observations, a tentative conclusion may now be reached, one which bears directly on the second criterion for tube-characteristic selection as brought out at the first of this chapter. The tube type to be selected for a mixer should have a large grid base, i.e., it should provide a relatively large span of grid-voltage variation in the negative-grid region before grid outoff is reached, compared to the peak-to-peak value of the largest total signal voltage to be applied, in effect, to the grid. The large grid base permits the use of a large amplitude, E_o, of local-oscillator voltage without violation of the requirement for class-A_1 operation as brought out earlier in this paper.

For triodes having a given value of amplification factor, μ, the larger the grid base desired, the larger the operating plate voltage required. It can be seen that this is true by assuming μ to be a constant in equation (6) which shows the total plate current of a triode to be proportional to a power of $(E_c + \frac{E_{b,T}}{\mu})$, E_c being the total instantaneous grid-to-cathode voltage and $E_{b,T}$ being the total instantaneous anode-to-cathode voltage. If the variable components of the grid and plate voltages are assumed to be zero, it is seen that the plate current becomes zero at that value of grid-bias voltage for which

$$E_c = - \frac{E_{b,T}}{\mu}, \tag{45}$$

which is cutoff bias. Conversely,

$$E_{b,T} = \mu \ (-E_c), \tag{46}$$

which illustrates that, for a given value of μ, a relatively large grid base can be obtained only in a tube* which permits normal class-A_1 operation at a high value of d-c plate voltage. Since high plate voltages usually make for both low noise and low transit time, the only apparent serious objections to using a mixer requiring high plate voltage would be the increased hazard involved and the higher voltage ratings required of associated components and of the power supply required to furnish the voltage. At receiver levels, neither of these objections need usually be extremely difficult to overcome. Thus, it appears at this point not only that the tube type to be selected should

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*Remote-cutoff tubes excepted.

be, with respect to the d-c plate voltage, a high-voltage type, but also that the particular tube used should be operated at the highest plate voltage which is consistent with manufacturers' tube ratings and other design factors. This last statement is made, however, with the expectation that the large grid base so obtained will be maximally utilized by application of a correspondingly large local-oscillator voltage.

Unless some precautions, such as the ones just discussed, are taken to minimize the relative production of fourth-degree, second-order effects, the channel-to-channel spacing suggested by the second-degree considerations would by no means be adequate to prevent passage through the i-f pass band of significant sideband components generated in a channel adjacent to the desired channel. The suggested spacing, it will be recalled, was slightly more than $2\omega_{m,max}$ for an i-f bandwidth barely in excess of the same figure. The reason why the channel spacing would be insufficient can be realized when it is remembered that E_b, having a form represented by equation (43), is raised to the third power in (40) which illustrates the fourth-degree, second-order mixing for the undesired channel. This cubing of E_b in (40) results in sidebands of the converted undesired signal which are three times the width of the corresponding r-f sidebands before mixing. Hence, with the suggested channel spacing, one of the undesired triple-width sidebands could almost completely overlap the i-f pass band and would constitute interference. Fortunately, for many applications the interference thus generated would not be of a most objectionable nature because there would not ordinarily be anything within the i-f passed band to establish coherence* with the unwanted sideband energy. Thus, in an aural application, the interference would usually sound somewhat like "noise."

To attempt to avoid this type of interference by increasing spacing between channels is not an attractive idea for two reasons. First, spectrum space would be consumed extravagantly. Second, terms of higher degree than the fourth would produce interference sidebands of still greater width; thus still greater channel-to-channel spacing would be required. Eventually the increasing of channel spacing could be carried to an extreme such that the r-f selectivity could satisfactorily reject all but the desired channel or else that the higher-degree terms not taken into account would be of insignificant magnitude. The need for spectrum conservation is emphasized in the "Introduction" to this thesis.

A number of the fourth-degree frequency components not already discussed can be treated with dispatch. The frequencies $2(\omega_a - \omega_o)$ and $2(\omega_b - \omega_o)$ either equal or approximate twice the intermediate frequency and so can be readily rejected by the i-f selectivity. The frequency $2(\omega_a - \omega_b)$ could cause some difficulty, especially since the associated amplitude factor is $\frac{3}{4} E_a^{\,2} E_b^{\,2}$. With an adjacent-channel separation (on centers) of $2\omega_{m,max}$ and with ω_a and ω_b some positive integral number, n, of channels apart, then

$$2\,(\omega_a - \omega_b) = 2n\,(2\omega_{m,max}). \tag{47}$$

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*A definitive discussion of coherence is outside the scope of this thesis but is adequately treated in Goldman's Frequency Analysis, Modulation and Noise.[16]

The greater the r-f selectivity, the smaller will be the maximum value which n can attain, for signals which are more than a few channels away from the desired channel will be prevented from reaching the mixer. Now, with E_a and E_b each of the form given by equation (43) and with the maximum modulation frequency, $\omega_{m,max}$, simultaneously present in both E_a and E_b, the amplitude factor $\frac{3}{4} E_a^{\ 2} E_b^{\ 2}$ can give rise to sideband frequencies as much as $4\omega_{m,max}$ away from, and on either side of, the center frequency, $2(\omega_a - \omega_b)$. Since adjacent-channel frequency separation is ordinarily far less than the intermediate frequency, any constraint of interest here is one necessitated by $2(\omega_a - \omega_b)$ and its associated upper sideband approaching the i-f pass band from the low-frequency side. The highest frequency involved here is the sideband frequency,

$$2n\ (2\omega_{m,max}) + 4\omega_{m,max} = 4\omega_{m,max}\ (n + 1). \tag{48}$$

Interference will begin to occur when this highest frequency just equals the lower edge of the i-f pass band, i.e., when

$$4\omega_{m,max}\ (n + 1) = \omega_{i-f} - \omega_{m,max}. \tag{49}$$

Thus, this interference can be avoided in design by choosing an i-f center frequency such that, to a good approximation,*

$$\omega_{i-f} \mathrel{\overline{>}} \omega_{m,max}\ (4n + 5). \tag{50}$$

Next, consider the fourth-degree frequency components $3\omega_a - \omega_o$, $3\omega_b - \omega_o$, $3\omega_a - \omega_b$, and $3\omega_b - \omega_a$. It is apparent that each of these will fall far above the intermediate frequency and so will be rejected.

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*See footnote, p. 45

The frequencies $3\omega_o - \omega_a$ and $3\omega_o - \omega_b$ require a little more precaution. If, as has been stipulated earlier, ω_a and ω_b each is approximately equal to the desired r-f frequency, $\omega_{desired}$, the two difference frequencies just above can each be approximated as $3\omega_o - \omega_{desired}$. Interference will then occur if this last frequency is in the vicinity of the intermediate frequency [see equation (32)], i.e., if

$$3\omega_o - \omega_{desired} \approx \omega_{desired} - \omega_o = \omega_{i-f}$$

or

$$\omega_o \approx \frac{1}{2}\,\omega_{desired}. \tag{51}$$

Thus, interference from the frequencies $3\omega_o - \omega_a$ and $3\omega_o - \omega_b$ can be avoided if the local-oscillator frequency is distinctly different from one-half the frequency to which the r-f input selective circuits are tuned. The reader will undoubtedly recognize immediately a further danger which would arise if the local-oscillator were to run at or near the particular frequency just mentioned. Second harmonics of the local-oscillator frequency would fall within the r-f pass band. Unless these harmonics were prevented (as by effective shielding and by filtering of power-supply leads) from reaching the r-f bandpass circuits, a type of interference would result. Actually, this particular value of local-oscillator frequency is usually easy to avoid and is, in fact, almost never encountered in actual practice.

It having been previously assumed that the local-oscillator frequency is to be considerably higher than the intermediate frequency, the fourth-degree frequencies $2\omega_o + \omega_a - \omega_b$ and $2\omega_o + \omega_b - \omega_a$ will each be easily rejected by the i-f selectivity. This is true because each is approximately equal to twice the local-oscillator frequency and hence is quite far from the i-f pass band.

If the frequency $\omega_a + \omega_b - 2\omega_o$ be rewritten as $(\omega_a - \omega_o) + (\omega_b - \omega_o)$, it is seen that this frequency is very nearly equal to $2\omega_{i-f}$ and hence will be rejected by the i-f selectivity. It is also clear that the frequencies $2\omega_a + \omega_o - \omega_b$, $2\omega_a + \omega_b - \omega_o$, $2\omega_b + \omega_o - \omega_a$, and $2\omega_b + \omega_a - \omega_o$ can be easily filtered out by the i-f selectivity, for each of these frequencies is definitely in the r-f region.

The two fourth-degree frequency components of equation (29) which have not yet been discussed are

$$\frac{3E_a{}^2E_oE_b}{2} \cos\,(\omega_o + \omega_b - 2\omega_a)t \tag{52}$$

and

$$\frac{3E_b{}^2E_oE_a}{2} \cos\,(\omega_o + \omega_a - 2\omega_b)t. \tag{53}$$

At once it is evident that the two have the same form, the only difference being an interchange between the a and the b subscripts. It is also apparent that if either amplitude factor, E_a or E_b, is identically zero, both terms vanish. It will now be shown that if the foregoing components are nonzero, either can readily represent interference which falls at the intermediate frequency and which therefore cannot be eliminated by such means as controlling the i-f selectivity. The type of interference which results can be classed as intermodulation.*

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*See the discussion of intermodulation in the Introduction.

Consider the first of the two components, shown by (52). Since it is a cosine function and since the cosine of an angle is the same as the cosine of the negative of the angle, the angular frequency expressed can be written as $2\omega_a - \omega_b - \omega_o$. Suppose that ω_a differs from the desired frequency, $\omega_{desired}$, by some arbitrary amount, $\overline{\Delta\omega}$, such that

$$\omega_a = \omega_{desired} \pm \overline{\Delta\omega}. \tag{54}$$

It is of interest to find, in terms of $\omega_{desired}$ and $\overline{\Delta\omega}$, what value of ω_b will combine with this particular value of ω_a to make the composite frequency expression, $2\omega_a - \omega_b - \omega_o$, equal the intermediate frequency, ω_{i-f}, expressed by $\omega_{desired} - \omega_o$, as shown in equation (32). Thus,

$$\begin{aligned} 2\omega_a - \omega_b - \omega_o &= 2(\omega_{desired} \pm \overline{\Delta\omega}) - \omega_b - \omega_o \\ &= (\omega_{desired} - \omega_o) + \omega_{desired} \pm 2\overline{\Delta\omega} - \omega_b \\ &= \omega_{i-f} + (\omega_{desired} \pm 2\overline{\Delta\omega} - \omega_b). \end{aligned} \tag{55}$$

It is clear that

$$2\omega_a - \omega_b - \omega_o = \omega_{i-f} \tag{56}$$

under the condition that

$$\omega_{desired} \pm 2\overline{\Delta\omega} - \omega_b = 0. \tag{57}$$

Hence, the value of ω_b which will cause interference is given by

$$\omega_b = \omega_{desired} \pm 2\overline{\Delta\omega}. \tag{58}$$

Comparison of equations (54) and (58) shows that the production of the fourth-degree, fourth-order intermodulation interference output from the mixer to the i-f pass band requires only the presence of two undesired signals, one spaced, frequency-wise, twice as far from the desired frequency as the other, both undesired frequencies falling on the same side of the desired signal--either above or below.

It should be stressed that this intermodulation interference can occur even in the absence of an actual signal at the desired frequency. Moreover, it should be emphasized that the brief analysis just made shows the relationship among three discrete frequencies of zero bandwidth when two of the three produce an interference component specifically at the third. The matter is complicated by one or both of the interfering signals being comprised of bands of frequencies, as by carriers and their associated sidebands. In addition, the discrete intermediate center frequency is by no means the only frequency at which a resultant produced by signals centered at ω_a and ω_b can cause interference. Rather, interference will exist if resultants fall at any or many of all the possible frequencies within the i-f pass band. The discrete-frequency approach is used here, though, for clarity as well as for simplicity.

It was pointed out previously that the two fourth-order intermodulation components, (52) and (53), have precisely the same algebraic form, the only difference being an interchange of the subscripts a and b. Therefore, the second component might be expected to represent an intermodulation resultant in the same fashion as the first. The primary difference of interest here is that the resultants fall at distinctly different frequencies. The difference is quite enough that even if a normal i-f pass band is blanketed by interference from one of the components, interference from the other can be rejected by i-f selectivity in the usual case. That this is so can be shown in the following way.

Imagine that the mixer is part of an amplitude-modulation (A-M) system which provides for channels having a uniform spacing, on centers, of $2\omega_{m,max}$, and let this spacing be designated as $\overline{\Delta\omega}$. Let the frequency, ω_o, of the local oscillator be such that an incoming r-f signal at the desired carrier frequency, $\omega_{desired}$, would be converted to an i-f signal having a carrier at $\omega_{i-f} = \omega_{desired} - \omega_o$. However, suppose that no desired signal is present but that two undesired modulated signals, A and B, on an adjacent and the next alternate channel, respectively, are present at the mixer r-f input. The respective carrier frequencies, ω_a and ω_b, satisfy the previously-determined condition for producing a fourth-order intermodulation resultant at ω_{i-f}, the center of the i-f band, because in the present situation

$$\omega_a = \omega_{desired} \pm \overline{\Delta\omega} \tag{59}$$

and

$$\omega_b = \omega_{desired} \pm 2\overline{\Delta\omega}. \tag{60}$$

Thus, when (59) and (60) are substituted into the expression, $2\omega_a - \omega_b - \omega_o$, for the frequency of the first fourth-order intermodulation component [see (52)], the carrier frequency of the resultant becomes

$$\begin{aligned} 2(\omega_{desired} \pm \overline{\Delta\omega}) - (\omega_{desired} \pm 2\overline{\Delta\omega}) - \omega_o \\ = (\omega_{desired} - \omega_o) \\ = \omega_{i-f}. \end{aligned} \tag{61}$$

But when (59) and (60) are substituted into the expression, $2\omega_b - \omega_a - \omega_o$, for the frequency of the second fourth-order intermodulation component [see (53)], the carrier frequency of this resultant is

$$\begin{aligned} 2(\omega_{desired} \pm 2\overline{\Delta\omega}) - (\omega_{desired} \pm \overline{\Delta\omega}) - \omega_o & \\ = (\omega_{desired} - \omega_o) \pm 3\overline{\Delta\omega} & \\ = \omega_{i-f} \pm 3\overline{\Delta\omega}. & \end{aligned} \tag{62}$$

Therefore, the carrier frequency of the second fourth-order intermodulation component is displaced from the center of the i-f band by the amount $3\overline{\Delta\omega}$, which equals $6\omega_{m,max}$ for the present hypothetical conditions. In actual practice $3\overline{\Delta\omega}$ would ordinarily equal somewhat more than $6\omega_{m,max}$, because practical allocations usually provide a suitable guard band between adjacent edges of adjacent channels. Be that as it may, an idealized i-f response will pass all frequencies within $\omega_{m,max}$ of its center at ω_{i-f} and will reject all frequencies farther displaced than $\omega_{m,max}$ from ω_{i-f}. Hence the carrier of the second fourth-order intermodulation resultant will be rejected.

If each of the two undesired signals, A and B, is modulated with frequencies as high as $\omega_{m,max}$, sidebands can extend for as much as $3\omega_{m,max}$ each side of the rejected carrier. That this is true can be seen by examining the amplitude factor of (53) and allowing E_o to be a constant and E_a and E_b each to be amplitude-modulation functions of the form shown by equation (43). The closest edge of one of these interference sidebands will still be $2\omega_{m,max}$ or $\overline{\Delta\omega}$ away from the nearest portion of the idealized i-f response band. Hence, one of the two fourth-order intermodulation components may be rejected entirely by i-f selectivity.

It is now timely to examine specifically the various parts of (19), which, it is recalled, symbolizes the explicit expansion, through the fourth-degree term, for the mixer plate current. On the basis of previous argument, let it be presumed desirable to make the second-degree term as large as possible but to keep to a minimum, for all terms, the interference which cannot be eliminated or minimized by means already suggested. It is of interest to consider the mixer a source of i-f voltage having an alternating plate voltage represented by

$$\begin{aligned} e_p &= e_1 + e_2 + e_3 + e_4 + \cdots \\ &= \sum i_1 r_p + \sum i_2 R_2 + \sum i_3 R_3 + \sum i_4 R_4 + \cdots . \end{aligned} \tag{63}$$

If $\sum i_3 R_3$, $\sum i_4 R_4$ and all the remaining summations higher than $\sum i_2 R_2$ could each be made zero, by the manipulation of either the i's or the R's, then no interference would remain which could not be controlled by means previously discussed. Hence, the possibility appears that the load resistance provides an additional controllable variable which might be made zero at all or most frequencies arising from the terms of (63) which are of higher degree than the second. The extent of, and the number of possibilities for, interference would thus be eliminated or at least minimized.

At the outset, on the other hand, it is seen that if e_2 is to be made large, R_2 must be made large compared to r_p. However, the fact that mixer output is desired only within a small spectral segment centered about one and only one frequency (the intermediate frequency) provides good reason for making R_2 zero at all frequencies except this one limited region, if it is possible to do so. By the same token, if e_2 is to be made as large as possible, e_1 should also be made as large as possible, for e_2 is proportional to e_1^2. Since e_1 is proportional to $\frac{r_p}{r_p + R_1}$, it appears at this point that R_1 should be made zero or at least very small compared to r_p. With the chosen intermediate frequency well displaced from the frequency of any r-f signal reaching the mixer, as has been assumed earlier, it is quite consistent to make R_1 small and R_2 large, both compared to r_p, within the r-f and the i-f pass bands, respectively.

Now consider e_3. At most, e_3 gives rise to the frequencies encountered in expanding e_g^3, which include all the frequencies related to e_1 but not the intermediate frequency associated with e_2. Therefore, all of e_3 can be effectively rejected by i-f selectivity. For this reason, it appears at this point that R_3 might be permitted to assume any convenient value consistent with the other R's. So far, this consistency amounts to R_3 being identical to R_1, because of the frequencies common to e_1 and e_3. Since e_1 and e_2 are required to be as large as possible and since e_3 otherwise is proportional to $\frac{R_3}{r_p + R_3}$, the zero or very small value which R_3 acquires from R_1 makes e_3 zero or a minimum.

An explicit expression for e_4 results from the multiplication of the fourth term of (19) by R_4. At most, the frequencies occurring in e_4 are those found by expanding e_g^4. Now, all the e's of higher order than the fourth (which would depend upon e_4 for their values) have been assumed negligible. Also, the only components of e_4 which cannot be eliminated by selectivity are those having frequencies in the immediate vicinity of the intermediate frequency. Furthermore, with suitable selection of the local-oscillator frequency and hence of the intermediate frequency according to the guides laid down previously, there will be no components of e_4 having frequencies common to e_1. It follows that R_4 may be made zero at all frequencies except the intermediate frequency. This designates R_4 as being equal to R_2. On this basis, R_4 is much larger than r_p for the narrow spectral region immediately around the intermediate center frequency and is zero at all other frequencies.

It is now seen that the passive plate-load resistance of the mixer considered as a voltage-producing device may consist of only two frequency-selective resistances, R_1 and R_2, for which the frequencies of nonzero values are mutually exclusive. This situation can be approximated physically by two parallel-tuned (anti-resonant) circuits placed in series with each other in the plate-to-cathode a-c return circuit. However, since R_1 is to be as nearly zero as it can practically be made even within the r-f pass band while permitting signal voltage to be applied in the plate circuit of the mixer, R_1 might well be simply the internal impedance, assumed resistive, of a low-impedance source. This source, of course, must provide all needed r-f selectivity, since in the case of R_1 being or approaching zero at all frequencies, little or no selectivity can be expected from R_1 as a function of frequency.

It will be noticed, above, that R_1 is not made zero but rather is considered to be a source impedance because of the necessity, in the low-interference plate mixer, for introducing the r-f signals into the plate circuit. Hence, the introduction of a nonzero value of R_1 appears unavoidable. All this is set forth here despite the fact that an introductory assumption to this analysis based upon a Taylor series was that "the plate circuit of the tube so described contains only a passive load resistance R(f), or simply R, a function of frequency." [See text immediately preceding equation (18).] Thus, the suggestion of introducing a signal source into the plate circuit may at first appear inconsistent with the analysis. However, if the generator voltage is referred to the grid while the generator internal resistance is thought of as remaining in the plate circuit, the analysis is still valid and applicable. It is recognized that the reference of the signal voltage to the grid circuit, for the class-A_1 conditions with constant μ as prescribed, can be made by merely dividing the applied signal voltage by μ and setting it into the grid circuit.

Conversely, the series analysis so far made is based on the assumption of both signal and local-oscillator voltage sources being in series in the grid circuit, but it is equally valid if the signals are applied in the plate circuit instead, provided that their amplitudes are μ times as large as before and that all circuit impedances remain the same for the two cases.

The actual plate-mixer circuit might be represented as in Figure 1, with the local-oscillator voltage source introduced between grid and cathode, with the signal source introduced into the plate circuit, and with the useful load also constituting a portion of the plate circuit. However, as far as concerns equation (17), as abbreviated by the summation

$$i_p = \sum_{m=1}^{\infty} \frac{1}{m!} \cdot \frac{\partial^{m-1} g_m}{\partial E_c^{\,m-1}} \left(e_g + \frac{e_p}{\mu}\right)^m \quad , \tag{64}$$

an exactly equivalent circuit would be as shown by Figure 2, all impedances being left as before but with the alteration that the total input signal voltage as it appears between plate and cathode, reduced in amplitude by a factor of μ and represented as an ideal voltage generator, would be removed from the plate circuit and placed in series with the local-oscillator voltage generator. In either case, the parenthetical quantity $(e_g + \frac{e_p}{\mu})$ could be shown as

$$e_{osc} + \frac{e_{sig}}{\mu} - \frac{1}{\mu}\, e\left[i_p(f),\ R(f)\right] \tag{65}$$

and equations (17) through (20) would apply equally well. Hence, these equations, which were advanced on the assumption of only passive impedance in the plate circuit, also adequately describe the plate mixer so long as the conditions of the equivalence are kept in mind.

Further consideration of (19) is called for since a way still needs to be found to establish a suitable operating point among the infinity of points along the selected static transfer characteristic. One approach is to attempt to force the fourth-degree term of (19) to zero or to a low minimum relative to the second-degree term. This is a valid procedure since this term can give rise to interference components produced in the manner previously discussed in connection with $e_g^{\,4}$, some of these components not being removable by conventional r-f and/or i-f selectivity. Yet the desirability of keeping the second-degree term high must be kept in mind.

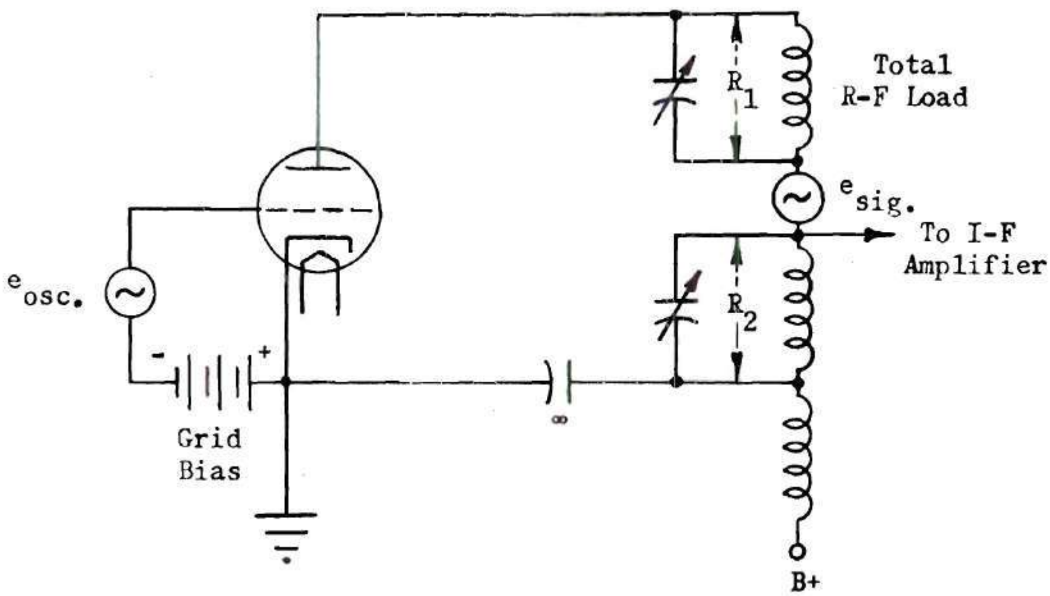

Figure 1. Triode Plate Mixer (Simplified Schematic)

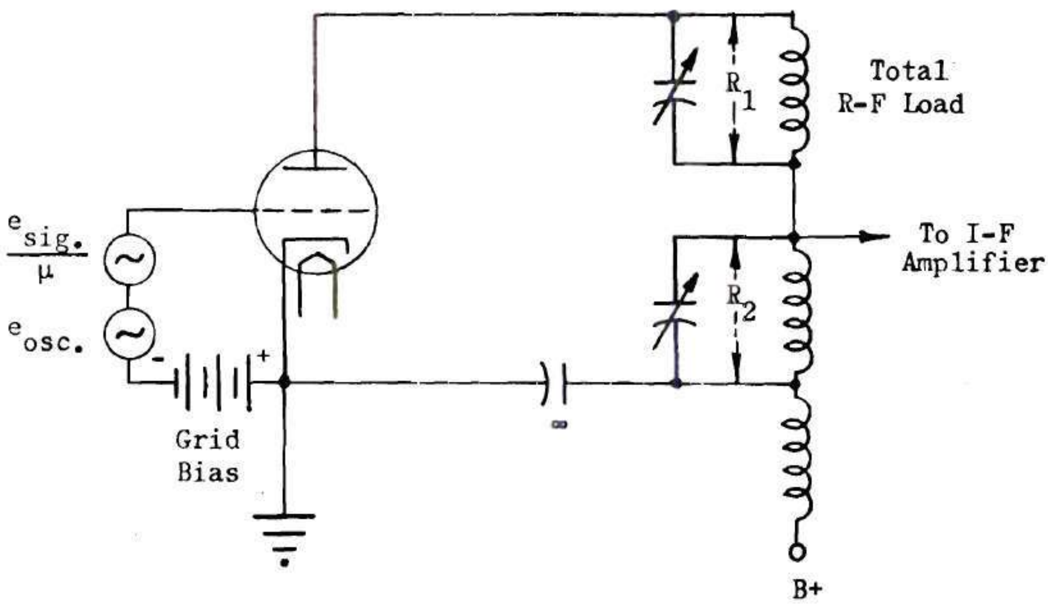

Figure 2. Grid-Mixer Equivalent of Triode Plate Mixer
(Simplified Schematic)

First, imagine that the fourth-degree term is multiplied by R_4, so that it is converted to the corresponding output voltage. Since R_4, as used here, is actually an abbreviation of a functional notation which is equivalent to R_2, it is valid to replace R_4 by R_2. Thus, since R_2 is zero at all relevant frequencies except within the i-f pass band, where it is very high compared to r_p, it is presently sufficient to manipulate just the numerator of the fourth-degree term with respect to the i-f band only.

Now, examine this numerator, starting at the right and working toward the left. The frequencies of the components resulting from e_2^2 are approximately zero and twice the intermediate frequency. Since e_2 now represents only the i-f pass band, these frequencies can be quickly checked by the substitution into e_2^2 of a cosine function corresponding to a carrier at the intermediate frequency. Since e_2^2 will produce no component at, or very near, the intermediate frequency, it will make no contribution to the fourth-degree term of (19).

The next expression in the numerator is

$$- \frac{1}{2!\mu g_m} \frac{\partial g_m}{\partial E_c} (2e_1e_3). \tag{66}$$

Consider e_1e_3 (the other factors being irrelevant here). If this product has no nonzero component within the i-f band, its effect here is zero. Substitution for e_3 in e_1e_3 gives

$$e_1e_3 = \frac{\dfrac{1}{3!\mu^2 g_m} \dfrac{\partial^2 g_m}{\partial E_c^{\,2}} e_1^{\,4} - \dfrac{1}{\mu g_m} \dfrac{\partial g_m}{\partial E_c} e_1^{\,2} e_2}{r_p + R_3} \cdot R_3 \, . \tag{67}$$

Since e_2 is a function of e_1^2, then, at most, the frequencies produced by (67) are those found in e_g^4. If all design suggestions up to this point are taken into account, R_3 will be zero at each of these frequencies. Therefore, e_1e_3 will be zero and may be discarded.

The next expression to the left in the numerator of the fourth-degree term of (19) is

$$- \frac{3}{3!\mu^2 g_m} \frac{\partial^2 g_m}{\partial E_c^2} e_1^2 e_2 \tag{68}$$

and the last expression to the left is

$$\frac{1}{4!\mu^3 g_m} \frac{\partial^3 g_m}{\partial E_c^3} e_1^4 \quad . \tag{69}$$

It might reasonably be questioned whether these two terms, being of opposite sign, might be made to equal each other in magnitude and thus be made to cancel for all values of applied voltage, e_g. To demonstrate that the two cannot in general be equated, it is sufficient to show, for normal circumstances, that one of the two can produce an output at the intermediate frequency simultaneously with the i-f output of the other being zero.

Consider the following. The e_1^4 term is proportional to e_g^4, since e_1 is proportional to e_g. It has already been shown that e_g^4 can produce i-f output by intermodulation of two undesired signals, neither of which lies within the desired channel. It has been brought out that this interference, once formed, cannot be eliminated by conventional approaches to selectivity. Yet if none of the two or more signals responsible for the interference lies within the desired channel at radio frequencies, then none is necessarily passed by the i-f response after simple second-degree mixing has taken place. Therefore, e_2 might very well be zero, since, at this stage, e_2 can be taken to represent merely the i-f response, one channel in width, to a signal originally falling within the desired r-f channel. It follows, then, that the e_1^4 term, (69), can readily produce i-f components under ordinary conditions for which the $e_1^2e_2$ term, (68), is zero. Hence, (68) and (69) should not be equated.

Another reasonable approach might be to minimize the algebraic sum of the two terms, (68) and (69). An examination of this approach reveals that in a given situation it offers a possibility for minimizing cross-modulation. In a typical case, however, this minimum is not of a different order of magnitude from the value to be obtained by still another approach to be examined shortly. On the other hand, the minimizing of the algebraic sum of (68) and (69) has two disadvantages. First, this method involves an excessive number of graphical steps which are quite laborious and which are subject to considerable cumulative error even though done with care. Sufficient inaccuracy ordinarily exists in published graphical data on tubes to keep the method from having an advantage of reliability even if it is used. Second, this method permits the continuing existence of fourth-degree intermodulation components which can theoretically be completely eliminated by the method yet to be proposed. Moreover, the operating point selected by this method is usually so near the point found by the method yet to be proposed that in a practical case one point can readily serve as a guide for locating the other by circuit adjustment, if such should be desired. However, for the benefit of any who might be interested in using this method, the operating point established by it has been found to be the point at which

$$\frac{\partial^3 g_m}{\partial E_c{}^3} = \frac{2}{g_m} \frac{\partial g_m}{\partial E_c} \frac{\partial^2 g_m}{\partial E_c{}^2} . \tag{70}$$

If more than one such point exists in the negative-grid region, the one nearest the center of the grid base should ordinarily be used in the interest of permitting the largest possible local-oscillator voltage swing within the bounds of class-A_1 operation. This selection also is likely to correspond to a higher value of the ratio $\frac{\partial g_m}{\partial E_c} / g_m$ than the other possible choices made on the same basis, so that the purpose of obtaining maximum practical conversion gain* is usually served.

It now seems desirable to attempt to force either (68) or (69), or both, to zero separately and, if possible, to minimize separately either which is not made zero. The factors in (68) and (69) which can be permitted to be zero are the partial derivatives of g_m with respect to E_c. Since

$$\frac{\partial^3 g_m}{\partial E_c{}^3} = \frac{\partial}{\partial E_c}\left(\frac{\partial^2 g_m}{\partial E_c{}^2}\right) , \tag{71}$$

*See equation (81) for conversion gain.

it can be seen that $\frac{\partial^3 g_m}{\partial E_c^{\,3}}$ will be zero when $\frac{\partial^2 g_m}{\partial E_c^{\,2}}$ has either a maximum, a minimum, or a zero-slope point of inflection. Triode characteristics are such that it is highly unlikely that $\frac{\partial^2 g_m}{\partial E_c^{\,2}}$ will itself be zero simultaneously with its having a maximum, a minimum, or a zero-slope point of inflection. In fact, it is unlikely that the zero-slope point of inflection will occur at all. Therefore, it may be reasoned that (68) and (69) cannot both be made zero for the general case of a nonzero applied voltage, e_g. Hence, a choice must be made.

To begin, it must be emphasized that to make either (68) or (69) zero will provide an easy practical way for deciding upon a correct operating point. In the first case the operating point on the selected transfer characteristic is determined by the value of negative grid bias at which $\frac{\partial^2 g_m}{\partial E_c^{\,2}}$ passes through its zero value. In the second case, corresponding to (69), the operating point on the selected transfer characteristic is determined by the value of negative grid bias at which $\frac{\partial^3 g_m}{\partial E_c^{\,3}}$ passes through zero. For either case, in the event that two such points exist in the negative-grid region, the one corresponding more nearly to the center of the grid base will ordinarily prove the more satisfactory, since it will permit the larger local-oscillator swing of the grid without risk of grid rectification and the accompanying over-all nonlinearities. And, as before, such a selection is likely to correspond to a value of the ratio $\frac{\partial g_m}{\partial E_c} / g_m$ which is a near maximum among the choices possible on the same basis and in the same general region. Hence, maximum practical conversion-gain possibilities are approached.*

It is pertinent here to note that, for a typical case, the bias determined by equating the $\frac{\partial^3 g_m}{\partial E_c{}^3}$ to zero has been found to be roughly midway between zero bias and cutoff. Thus it provides approximately a maximum local-oscillator swing consistent with class-A_1 operation.

Next, consider the fact that $e_1{}^4$ is more complex than $e_1{}^2 e_2$ and offers more possibilities for producing interference. The least that $e_1{}^4$ can do is to produce frequency components directly corresponding to those produced by $e_1{}^2 e_2$, since e_2 is a restricted function of $e_1{}^2$ and hence $e_1{}^2 e_2$ is a restricted function of $e_1{}^4$. Further, it must be recognized that the undesired components related to $e_1{}^2 e_2$ are present only if a signal is present on the desired channel, because e_2 will otherwise be zero. Also, these undesired components (corresponding to $e_1{}^2 e_2$) are largest when the desired signal is large and are small when the desired signal is small.

Now, these facts are important from an operational viewpoint, particularly in an application such as two-way communication. Their importance depends a great deal upon the particular situation and upon operator experience. To an experienced operator, the interference components (cross-modulation) associated with $e_1{}^2 e_2$ tend not to obscure knowledge of whether a desired signal is present and, to some extent, not to obscure the desired signal itself. Admittedly, though, this latter tendency is not to be relied upon.

However, it has previously been pointed out that $e_g{}^4$ and

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*See equation (81) for conversion gain.

hence e_1^4 can be responsible for interference (intermodulation) with or without the presence of a signal on the desired channel. This intermodulation interference may be quite large in amplitude if the undesired signals producing it are large, even though the desired signal may at the same time be very small or may not exist at all. Hence, the intermodulation components associated with e_1^4 tend to obscure both the desired signal itself and the knowledge of whether the desired signal is present.

On the basis of the foregoing arguments, it is elected here to select the operating point on the chosen transfer characteristic such that $\frac{\partial^3 g_m}{\partial E_c^3}$ equals zero so as to force e_1^4, and hence the fourth-degree intermodulation interference, to zero for all values of desired and undesired signal (within class-A_1 limits).

As this election is made, it is done with the realization that there can be no further independent control of the interference produced by the $e_1^2 e_2$ term except through control of the local-oscillator and the applied-signal voltages. Argument has been previously advanced favoring keeping the local-oscillator voltage high. Also, it is evident that the desired-signal voltage should be maintained at a high level if possible, since a large desired signal ordinarily can be received through a mixture of interference more easily than can a small signal. Certainly, it would be desirable to keep undesired signals at the lowest possible level, but reduction of undesired signals without consequent reduction of the desired signal becomes a matter of some kind of selectivity. At this stage, however, it is assumed that all practically obtainable selectivity has already been achieved and that the remaining undesired signals are those which have not been eliminated. Thus, there is, in effect, no further control obtainable over the fourth-degree interference, except for one consideration. This one exception involves the fact that excessively large values of local-oscillator and/or applied-signal voltages may cause higher-degree terms of the describing series expansion to be comparable to or larger than the second-degree term. These higher-degree terms include the nonzero portion of the fourth-degree term. Even though these terms may be completely negligible at ordinary voltages, if the coefficients are nonzero the terms cannot be neglected if the voltages are made sufficiently high. This is true because the terms each involve positive integral powers of the voltages. Voltages likely to be encountered in receiver work frequently will not be bothersome, especially if the tube used has a high amplification factor.

The effects of the uneliminated portion of the fourth-degree term--including the effects, mentioned just above, of input voltage magnitudes--upon production both of desired output and of interference may be demonstrated by a consideration of the two-signal case. For this consideration let $E_a \cos \omega_a t$ be the desired signal and $E_b \cos \omega_b t$ be an undesired signal, both at radio frequencies. It is not difficult to show that the complete fourth-degree voltage, under the constraints already imposed, can be written

$$e_4 = - \frac{\mu}{8 g_m^2} \frac{\partial g_m}{\partial E_c} \frac{\partial^2 g_m}{\partial E_c^2} \Big\{ (E_o^3 E_a + E_a^3 E_o \tag{72}$$

$$+ E_b^2 E_o E_a) \cos (\omega_a - \omega_o)t + E_a^2 E_b E_o \big[\cos (\omega_b - \omega_o)t$$

$$+ \cos (2\omega_a - \omega_o - \omega_b)t]\Big\} .$$

If only unmodulated-carrier conditions are considered, the components at angular frequencies $\omega_b - \omega_o$ and $2\omega_a - \omega_o - \omega_b$ will fall outside the i-f channel and so will be filtered out. If, for expedience in comparison, E_a and E_b are assumed equal, it is seen that the remainder of e_4 consists of three components, two of which have precisely the same form and the same magnitude. One of the two, involving $E_a{}^3E_o$, is classed as distortion. The other, involving $E_b{}^2E_oE_a$, is classed as cross-modulation. Either will serve as a basis for comparison with desired output; so for convenience the $E_a{}^3E_o$ component will be employed.

The third remaining component of e_4 is the one involving $E_o{}^3E_a$ at the desired intermediate frequency. Since E_o is ordinarily a constant, this component, with or without conditions of modulation, will in all essentials resemble desired output. Compare this component,

$$- \frac{\mu}{8g_m{}^2} \frac{\partial g_m}{\partial E_c} \frac{\partial^2 g_m}{\partial E_c{}^2} E_o{}^3E_a \cos (\omega_a - \omega_o)t, \tag{73}$$

for the equivalent grid mixer, with the second-degree desired-output term for the equivalent grid mixer, which simplifies to

$$\frac{\mu}{2g_m} \frac{\partial g_m}{\partial E_c} E_oE_a \cos (\omega_a - \omega_o)t. \tag{74}$$

Since the two are of opposite sign, it is clear that (73) might conceivably cancel (74), in which case no production of desired output by terms of degree lower than the fifth would occur. Or, (73) might swamp (74), so that the second-degree desired output would be negligible by comparison with the fourth-degree desired output. Evidently a comparison of the two for a typical assumed situation is in order.

For this comparison observe the ratio of the fourth-degree "desired" component, (73), to the second-degree term, (74). This ratio is

$$M = -\frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_o^{\,2} . \tag{75}$$

For a type 6J4 tube operated in the prescribed fashion, typical values may be read from Figure 3 and are as follows:

$$g_m = 4300 \times 10^{-6} \text{ mhos},$$

$$\frac{\partial^2 g_m}{\partial E_c^{\,2}} = 3700 \times 10^{-6} \ \frac{\text{mho}}{\text{volt}^2}$$

and

$$E_o = 3 \text{ volts (peak value).}$$

Here the ratio, (75), becomes approximately -1.94. Thus, for this example, the fourth-degree mixing takes precedence over the second-degree mixing. If the coefficients of terms of degree higher than the fourth which also tend to generate "desired" output are sufficiently small in this specific case that their contributions may be neglected, it is seen that the net desired output as a result of both the second- and fourth-degree terms is only nine-tenths what it would have been from the second-degree term alone. A peak value of 2.15 volts for E_o would have made the ratio -1.0 and the net desired output from the

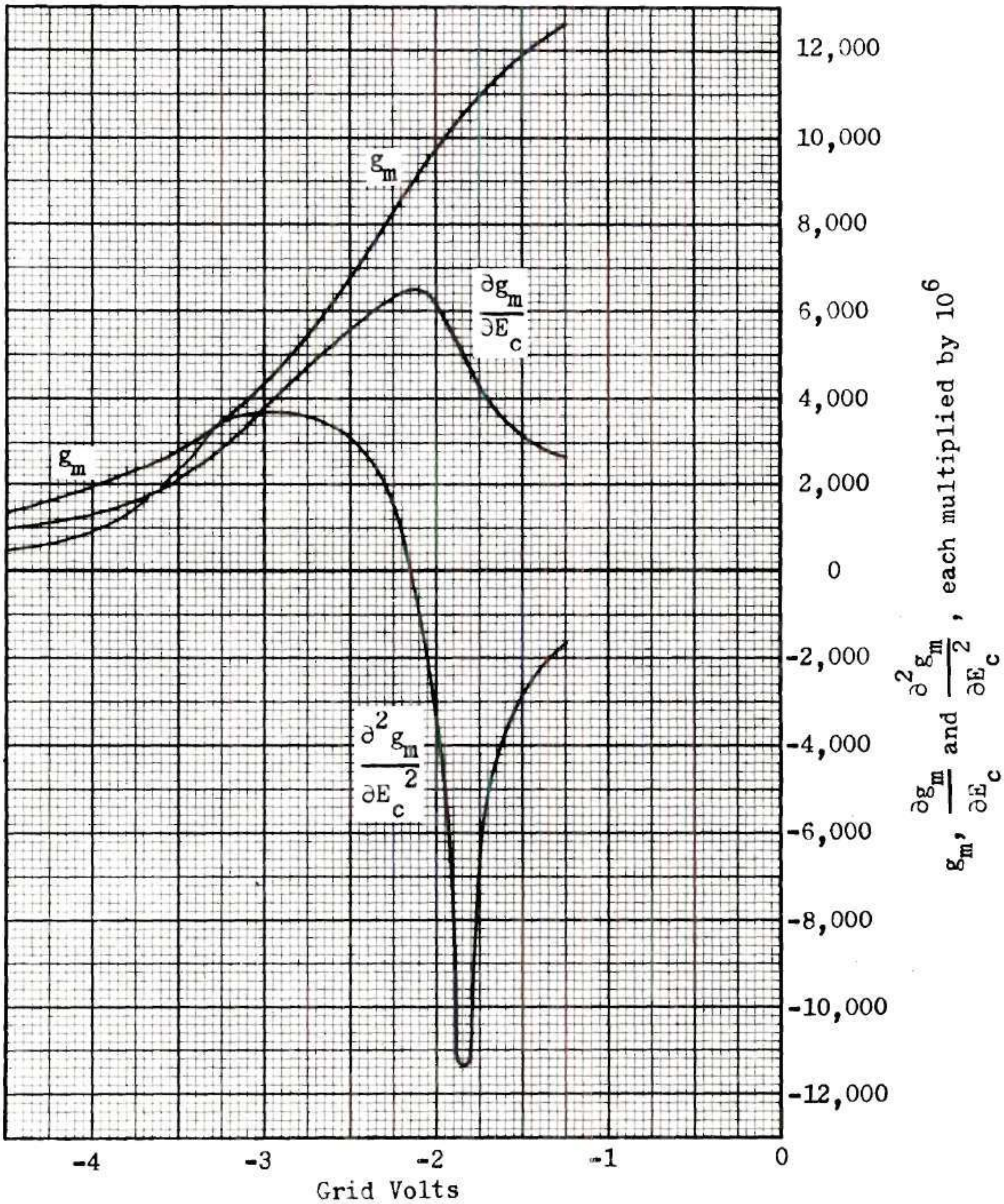

Figure 3. Operating-Point Determination for Maximum-Voltage Design of Typical Low-Intermodulation Plate Mixer

second- and fourth-degree terms would have been zero. Hence, care in selecting the local-oscillator voltage, E_o, appears important. Since a theoretical investigation of many of the higher-degree terms will usually be impractical, it is almost imperative that a design value for E_o be ascertained experimentally.

The minus sign in (73) and (75) suggests selection of a different operating point so as to make the negative term turn positive and hence to cause the second- and fourth-degree desired components to be additive. Thus, a point might be selected such that $\frac{\partial^3 g_m}{\partial E_c{}^3} = 0$, while $\frac{\partial^2 g_m}{\partial E_c{}^2}$ is negative and g_m and $\frac{\partial g_m}{\partial E_c}$ are both positive. However, at least two cautions are in order. First, E_o may be limited to a much smaller value than before by the limitations of class-A_1 operation. Second, one of the partials--say, $\frac{\partial^2 g_m}{\partial E_c{}^2}$ --may have such a rapidly changing value in the vicinity of this alternate point that stability both of conversion gain and of the minimizing of cross-modulation and intermodulation might be difficult to maintain in a practical situation. The stability of intermodulation rejection would re-enter the picture because the point at which $\frac{\partial^3 g_m}{\partial E_c{}^3} = 0$ might well become critical. However, the feasibility of using an alternate operating point in a given situation is certainly worth consideration. In this thesis the use of the alternate point is assumed to have been investigated and found impracticable.

Now, for observing the effects of the fourth-degree interference in the two-signal case, either of the two interference terms already noted may be used. It has already been decided to use the $E_a{}^3 E_o$ term as a matter of convenience. The ratio of this term to the algebraic sum of the second- and fourth-degree desired-signal terms will serve for the comparison. It is again assumed that all contributions from terms of higher degree than the fourth are negligible. The ratio of distortion to desired component for the equivalent grid mixer is easily shown to be

$$D_g = \frac{\frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_a^{\,2}}{\frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_o^{\,2} - 1} = \frac{M}{M + 1}\left(\frac{E_a}{E_o}\right)^2 . \tag{76}$$

For the plate mixer the effect of the applied-signal voltage is as if the voltage were divided by μ. Therefore, the ratio of distortion to desired-signal component for the plate mixer is

$$D_p = \frac{\frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_a^{\,2}}{\frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_o^{\,2} - 1}\left(\frac{1}{\mu^2}\right) = \frac{M}{M + 1}\left(\frac{E_a}{\mu E_o}\right)^2 . \tag{77}$$

For the same example as used above, the denominator, $M + 1$, of (77) has already been found to be approximately -0.94. The value of μ for the same situation is nominally 55. If E_a is assumed to be 1.0 volt, then, for the example the ratio of the distortion component to the desired-signal net component is 1 : 13,200; i.e., the distortion component is approximately 82.4 db below the desired-signal net component. If the signal voltage had been assumed equal to the local-oscillator voltage of 3.0 volts, peak, the distortion would still have been down approximately 63.3 db. The same figures would be obtained for the cross-modulation content if the undesired signal were taken equal to the desired signal, as suggested previously, and if no modulation existed on either carrier.

It is interesting to note that the importance of the fourth-degree term relative to the second-degree term is independent of the signal amplitude so far as the mixing action is concerned but not so far as the interference is concerned. It is also well to observe that the effect of μ is such as to make the plate mixer considerably freer of distortion and interference than the equivalent grid-mixer. The higher the degree of the term involved, the faster a large value of μ makes the interference contributed by that term negligible in the case of the plate mixer, as can be inferred now by reference back to (19).

It is now of interest to examine the conversion gain to be expected of the device. Again the terms of degree higher than the fourth will be assumed negligible. With this assumption as a beginning, a review of (75) discloses that the desired-mixing output voltage for the <u>equivalent grid mixer</u> is given by

$$V_{i\text{-}f,grid} = (1 + M)\, e_2 \tag{78}$$

where, at this stage, e_2 is understood to represent the desired i-f output produced by the second-degree term, and M is defined by (75). If, as before, the desired-channel r-f input voltage is described by $E_a \cos \omega_a t$, then the substitution of (74) and (75) into (78) gives

$$V_{i\text{-}f,grid} = \left(1 - \frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_o^{\,2}\right) \tag{79}$$

$$\cdot \frac{\mu}{2g_m}\frac{\partial g_m}{\partial E_c} E_o E_a \cos(\omega_a - \omega_o)t.$$

However, in the case of the plate mixer, the effect is as if the signal-voltage amplitude were reduced by the factor $1/\mu$. Hence, for the plate mixer,

$$V_{i\text{-}f,plate} = \left(1 - \frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_o^{\,2}\right) \frac{1}{2g_m}\frac{\partial g_m}{\partial E_c} E_o E_a \cos(\omega_a - \omega_o)t. \tag{80}$$

Thus, (80) is $1/\mu$ as great as (79). In either case, the actual desired-signal r-f voltage is the same--$E_a \cos \omega_a t$. Now, let the conversion gain be defined as the ratio of the magnitude of the desired-signal i-f output voltage to the magnitude of the desired-signal r-f input voltage. If this conversion gain be denoted by G_g for the grid mixer and G_p for the plate mixer, it can now be easily seen that

$$G_p = \frac{1}{\mu} G_g = \left(1 - \frac{1}{4g_m}\frac{\partial^2 g_m}{\partial E_c^{\,2}} E_o^{\,2}\right) \cdot \frac{1}{2g_m}\frac{\partial g_m}{\partial E_c} E_o \ . \tag{81}$$

It is noticed that the conversion gain is only $1/\mu$ as much for the plate mixer as for the equivalent grid mixer. On the other hand, it is interesting to note that for the plate mixer the conversion gain is independent of the amplification factor, μ.

It is seen that for a sufficiently small amplitude, E_o, of the local-oscillator voltage, the conversion gain simplifies to

$$G_p = \frac{1}{\mu} G_g \approx \frac{1}{2g_m} \frac{\partial g_m}{\partial E_c} E_o \ . \tag{82}$$

The realizable conversion gain, found according to equation (81), for the 6J4 example used earlier (with $\frac{\partial g_m}{\partial E_c} = 3800 \times 10^{-6} \ \frac{\text{mho}}{\text{volt}}$) is approximately -1.24 for the plate mixer. The negative sign merely expresses the superiority of the fourth-degree over the second-degree mixing and has no particular usefulness.

Previous discussion in connection with equations (75), (77) and (81) has pointed out the importance of the local-oscillator voltage, E_o, as a key parameter determining conversion gain and relative production of distortion and interference. It has also been strongly suggested that a design value for this voltage be found or checked experimentally. Even for the experimental determination, however, a brief study of the theory will be useful. To this end, consider the following discussion.

Assume that the desired operating point for the tube to be used has been located. For that particular point the transconductance, g_m, and all its partial derivatives are fixed, as is the amplification factor, μ. Now let

$$K_1 = \frac{1}{4g_m} \frac{\partial^2 g_m}{\partial E_c^{\,2}} \quad \text{and} \quad K_2 = \frac{1}{2g_m} \frac{\partial g_m}{\partial E_c} \ .$$

Then, by substitution of these into equations (77) and (81), the relative distortion for the plate mixer is given by

$$D_p = \frac{K_1 \left(\frac{E_a}{\mu} \right)^2}{K_1 E_o^2 - 1} \tag{83}$$

and the conversion gain is given by

$$G_p = (1 - K_1 E_o^2)(K_2 E_o) \; . \tag{84}$$

For a given value of signal voltage, E_a, the numerator of (82) is a constant, K_3, where

$$K_3 = K_1 \left(\frac{E_a}{\mu} \right)^2 \; .$$

The relative distortion now becomes

$$D_p = \frac{K_3}{K_1 E_o^2 - 1} \; . \tag{85}$$

Assume, for the moment, that K_1 is negative, in opposition to earlier assumptions regarding the probable selection of the operating point. It is at once apparent that the magnitude of the conversion gain becomes rapidly greater and the magnitude of the relative distortion (and similarly interference) becomes rapidly smaller as the local-oscillator voltage, E_o, is made greater. The earlier restriction to class-A_1 operation, however, places a relatively low upper limit on how large E_o can be made. As suggested earlier, this fact together with probable operational instability at the operating point selected to make K_1 negative tend to rule out the probability of the operating point being so chosen.

Therefore, assume next that K_1 is positive. Evidently the conversion gain is zero and the relative distortion or interference infinite when

$$K_1 E_o^2 = 1 \, , \tag{86}$$

i.e., when

$$E_o = \pm \frac{1}{\sqrt{K_1}} \, . \tag{87}$$

Also, gain, distortion and interference can all be seen to be zero when $E_o = 0$. Now let the conversion gain, equation (83), be maximized with respect to E_o by the standard procedure of differentiating (84) and equating the derivative to zero. Thus,

$$\frac{dG_p}{dE_o} = K_2 \, (1 - 3K_1 E_o^{\,2}) = 0 \, . \tag{88}$$

Upon the assumption that $K_2 \neq 0$, it is now found that the conversion gain, G_p, as a function of the local-oscillator voltage, E_o, passes through a maximum absolute value when

$$E_o = \pm \frac{1}{\sqrt{3}} \frac{1}{\sqrt{K_1}} \approx \pm \, 0.58 \frac{1}{\sqrt{K_1}} \, . \tag{89}$$

Equation (88) shows a magnitude roughly halfway between the two magnitudes of E_o for which the conversion gain becomes zero according to the assumptions made earlier. The conversion gain at this value of oscillator voltage is found to be

$$G_{p,max} = \pm \frac{2}{3\sqrt{3}} \frac{K_2}{\sqrt{K_1}} , \tag{90}$$

the plus-or-minus sign indicating the same sequence as in (89). This same magnitude of conversion gain also occurs when

$$E_o = \pm 1.155 \frac{1}{\sqrt{K_1}} , \tag{91}$$

as found by substituting equation (90) into equation (84) and solving for E_o. For all magnitudes of E_o greater than that indicated by (90) the conversion gain continues to increase rapidly.

The fact that the conversion gain changes quite rapidly with changing local-oscillator voltage in this latter region warrants emphasis on three points. First, it is highly important that once E_o exceeds $1.155 / \sqrt{K_1}$, it should be made as large as possible, within the limits of class-A_1 operation, if maximum conversion gain is to be realized. Second, the constancy of the magnitude of E_o must be made excellent in this region if stable conversion gain is to be obtained. Third, to the extent that K_1 and K_2 fail to remain constant because of fluctuating tube characteristics, the gain will vary. However, equation (83) shows that this variation will not be extremely large because only the first powers of K_1 and K_2 enter into the conversion-gain equation and because the selection of the operating point suggested earlier was such that both K_1 and K_2 are not likely to vary greatly.

It may be seen from the foregoing that if maximum conversion gain and maximum rejection of interference and distortion are desired, the peak magnitude, E_o, of the local-oscillator voltage should be maintained as great as possible so long as it is greater than the value indicated by (90), yet still within the limits of class-A_1 operation. On the other hand, if stability of operation is a prime consideration even at the expense of a sacrifice in conversion gain and in rejection of interference and distortion, then E_o should have the value indicated by (88).

For the 6J4 example cited earlier it was assumed that the local-oscillator voltage excursions were as great as class-A_1 conditions would permit, i.e., 3.0 volts, peak. It is of interest to compare the values of conversion gain and of rejection of cross-modulation (or distortion) for that example with those obtained under the same circumstances except for E_o being set, as indicated by (88), for best stability of operation. In this latter case E_o becomes approximately ± 1.24 volts, peak. The comparison may be made by inspection of Table 1, below. The distortion (or cross-modulation) values are given in the units of db below the desired-signal output voltage.

Table 1. Effect of Local-Oscillator Voltage on Conversion Gain and on Distortion (or Cross-Modulation)

Local-oscillator voltage (peak volts)	Desired signal applied (rms.volts)	Conversion gain	Distortion or cross-modulation (db)
± 3.00	3.0	± 1.24	- 63.3
± 1.24	3.0	± 0.37	- 60.3
± 3.00	1.0	± 1.24	- 82.4
± 1.24	1.0	± 0.37	- 79.4

CHAPTER IV

EXPERIMENTAL TESTS ON A SIMULATED PRACTICAL PLATE MIXER

An exhaustive experimental study of the plate mixer could, by itself, easily constitute sufficient material for a thesis. The exigencies of time have not permitted such a study here or even an attempt to uncover all the discrepancies in the developed theory. Rather, the principal object here is to disclose whether experiment agrees in trend with the theory.

The following gives in brief the specific information which was sought. First, it was desired to find, for constant values of heater and plate-to-cathode d-c voltages and of local-oscillator swing, whether there existed a distinct value of grid bias at which the production of fourth-order intermodulation was at a minimum. A comparison of this value, if found, with the anticipated value would be of interest. Second, with the heater and the plate-to-cathode voltages still held constant and with the grid bias also held constant at the value corresponding to the intermodulation minimum just mentioned, it was desired to determine the characteristic of conversion gain versus local-oscillator voltage amplitude. A comparison of this characteristic with the one found theoretically by neglecting higher-degree effects would likewise be enlightening.

For these purposes an experimental setup was made as shown below in Figure 4. The type 6J4 tube, having a nominal amplification

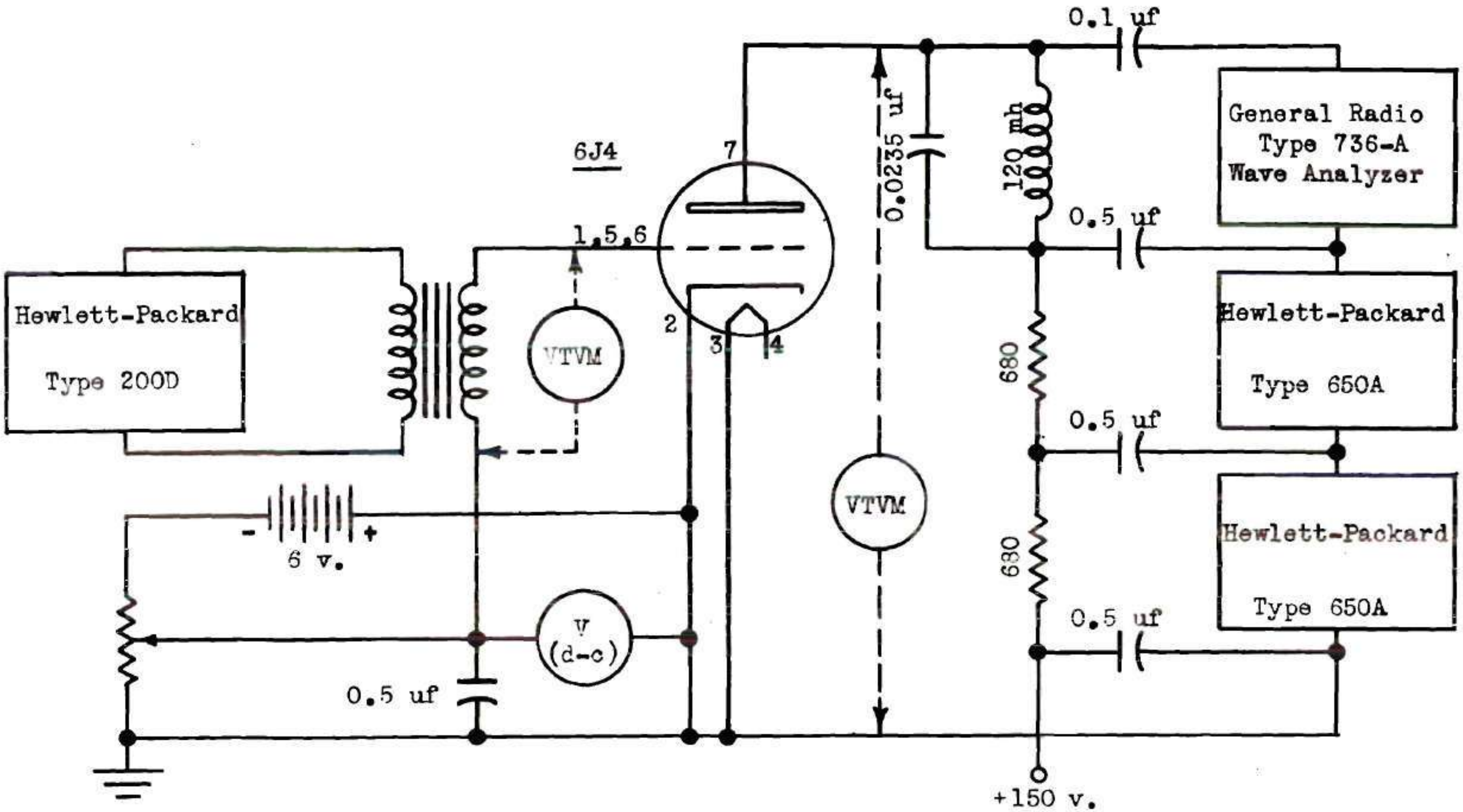

Figure 4. Simulated Practical Triode Plate Mixer

factor of 55, was chosen for test because of its wide present-day use in the general vhf-uhf field and because it appears to be one of the best presently available types for use specifically in the low-interference plate mixer. The more recent but less familiar type 6AM4, having a nominal amplification factor of 85, would probably be even better than the 6J4, although both are especially intended for uhf applications.

The particular signal generators and wave analyzer used were chosen for their dependability, low internal noise, and low internal intermodulation characteristics.

In the wiring of the circuit shown, all internal connections were carefully soldered with the exceptions of those to the tube. This soldering was a vital point because previous experience had revealed the possibility of serious errors being caused by even a single "cold-soldered" joint. The effort was made to keep the number of pressure connections, even to external pieces of equipment, to a minimum. Where pressure connections could not practically be avoided--specifically, to the test equipment terminals--these connections were tightly made.

Except for meters internal to the test equipment, all meters anywhere in the r-f circuit were removed before each reading of intermodulation. This was done to reduce the possibility of extraneous intermodulation being produced at pressure connections and in the diodes of meter probes.

From the diagram of Figure 4 it may be noticed that appreciable resistance was inserted in the plate circuit. Each of the two 680-ohm resistors, when shunted by the remainder of the circuit, provided an essentially correct termination for one of the signal generators. However, these terminations had no particular merit by virtue of providing matched loads for the generators. Rather, it was desired to isolate the signal generators from the d-c plate-voltage supply, and the resistance-capacitance coupling provided a convenient means for doing so. Because of the 600-ohm internal resistance of each generator being in shunt with the respective load resistor at radio frequencies, the net plate load resistance at the input frequencies was approximately 640 ohms--relatively low by comparison with a plate resistance of the order of 5000 ohms. Applied-voltage errors due to the resistors were eliminated by reading both the d-c and the a-c voltages directly between plate and ground.

It is true that the resistors created a small error in the total output i-f voltage as indicated on the wave analyzer because of the voltage divider effect. Actually, though, this error was negligible because the loaded impedance of the i-f tuned circuit with the wave analyzer for a load was upwards of 200,000 ohms, in comparison with which the remaining 640 ohms was insignificant. It is evident that the a-c load resistance at the intermediate frequency was high compared to the plate resistance. Thus, the test circuit provided the essentials stipulated by the theory.

The following is an outline of the procedure followed in making a representative measurement of intermodulation. The output voltage of each signal generator, including the one simulating the local-oscillator in the grid circuit, was set to its proper value with each of the others attenuated to essentially zero output. Then, with all generators running at proper output, the desired value of grid bias voltage was obtained by adjustment of the 1000-ohm bias potentiometer in the grid circuit. After this the plate-to-cathode voltage was adjusted to 150 volts. All voltmeters were then removed.

The "local-oscillator" frequency was 30.0 kc. and one "signal" generator was accurately set to 49.0 kc. so that the second "signal" generator at a frequency of 65.0 kc. would tend to cause a fourth-order intermodulation resultant at the 3.0-kc. intermediate frequency according to the relation

$$2f_1 - f_2 - f_{1.o.} = f_{i\text{-}f} \qquad (92)$$

from which

$$2 \times 49.0 - 65.0 - 30.0 = 3.0 \text{ kc.}$$

The wave analyzer, tuned to 3.0 kc., served as a sharply tuned voltmeter to indicate the magnitude of the intermodulation resultant voltage. This value was recorded.

The output of one of the "signal" generators was next attenuated to zero and the frequency of the other "signal" generator was changed to 33.0 kc. without a change in its output voltage. Thus, a desired-signal input was simulated. Again a wave-analyzer reading was made of the "i-f output" voltage. The ratio, expressed in db, of this output voltage to that produced by intermodulation was taken as the intermodulation rejection of the mixer.

In the case of the class-A_1 operated plate mixer, intermodulation measurements made according to the foregoing procedure have been found to give results essentially the same as those found by a less expeditious, though standard, procedure. In this latter procedure, which may be followed for present purposes if desired, the equal values of two interfering signals are found which give the same i-f output voltage as does a reference value of "desired" signal. The differences between the two methods may be inferred but are of no especial importance here.

The procedure for making a determination of conversion gain was essentially the same as the second step of the intermodulation measurement. Again the output of one "signal" generator was turned to zero while the output of the other was set to 1.0 volt, rms., at the frequency 33.0 kc. The "local oscillator," at the frequency 30.0 kc., was set to some specified value and a wave-analyzer reading, representing the 3.0 kc. i-f output voltage, was taken. The ratio, expressed in decimal form, of the i-f output voltage to the signal input voltage was taken to be the conversion gain.

Results of these tests are shown graphically in Figures 5 and 6.

Figure 5 shows the anticipated minimum in intermodulation as grid-bias voltage is varied. However, this minimum occurs at a negative bias slightly greater than 3.8 volts as compared to a predicted value of 3.0 volts. Experimental error might be suspected as a contributor to the discrepancy, but a careful check showed the experimental error to be negligible. On the other hand, the tests were made on a single tube, whereas the curves for prediction were derived from a manufacturer's composite data on many tubes. Not only do the manu-

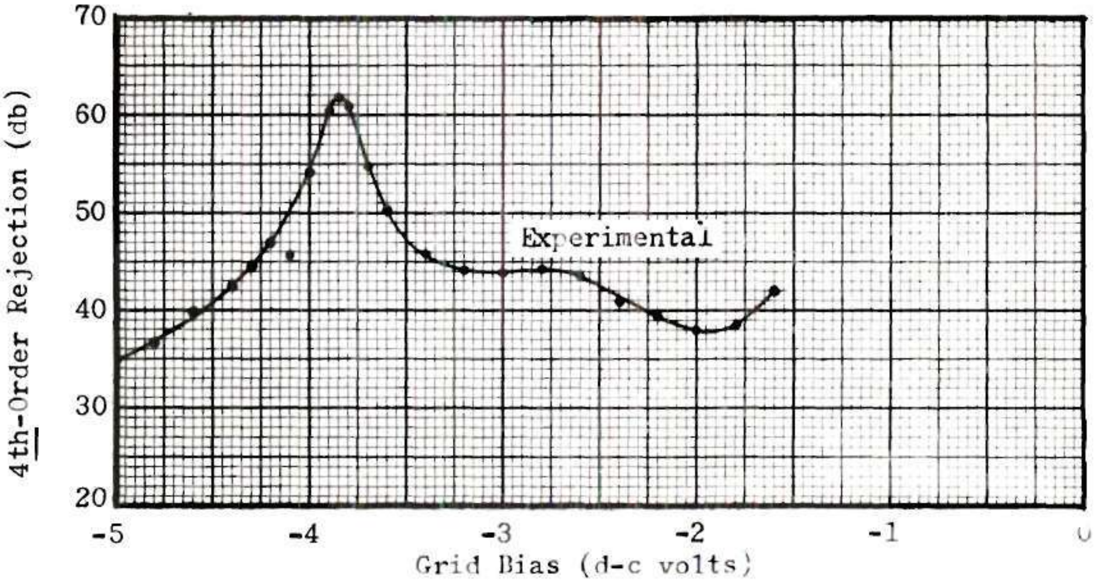

Figure 5. Fourth-Order Intermodulation Rejection versus Grid Bias for Typical Triode Plate Mixer

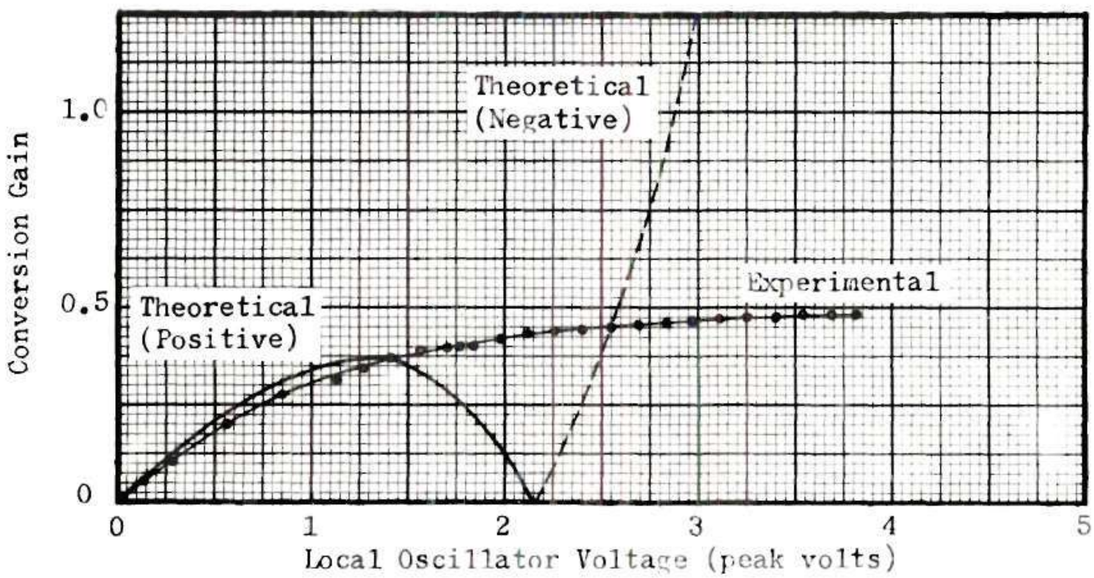

Figure 6. Conversion Gain versus Local-Oscillator Voltage Amplitude for Typical Triode Plate Mixer

facturer's data not necessarily correspond closely to any given tube, but also the curves of the several partial derivatives of the plate current are very difficult to derive accurately by ordinary graphical means. In addition, the higher-degree effects which were neglected in the theoretical development undoubtedly contribute some to a shift in the position of the minimum.

Figure 6 shows a curve of conversion gain versus local-oscillator voltage amplitude. At the lower values of local-oscillator voltage a good agreement exists between the experimental and the theoretical characteristics, but with increasing local-oscillator voltage a considerable divergence appears. There are at least two factors which might contribute to the difference.

Perhaps the most obvious factor is that the higher-degree terms which have been neglected in the theoretical development are not as negligible as they have been assumed to be for the higher values of local-oscillator voltage employed.

A second possible explanation is that the operating point, having been chosen at the grid-bias value experimentally giving the lowest <u>total</u> fourth-order intermodulation, was not necessarily at the value giving zero fourth-<u>degree</u> intermodulation. This is true because the minima of the higher-degree contributions are not known to occur all at the same operating point and because the relative contributions of the higher-degree terms for the larger values of local-oscillator voltage are not known. However, the point of zero fourth-<u>degree</u> intermodulation was the point on which the theoretical conversion-gain characteristic was based. Hence, a lack of agreement between the theoretical and the experimentally determined gain curves is understandable.

The experimentally determined conversion-gain characteristic has both a disadvantage and an advantage over the theoretical. The disadvantage is that the maximum realizable gain is markedly less than was anticipated. On the other hand, the advantage is that the operation is considerably less sensitive to fluctuations in local-oscillator voltage than was predicted.

The following miscellaneous observations were made which point toward avenues for further inquiry.

1. The noise level in the output appeared to be largely a function of the noise contained in the local-oscillator voltage.

2. The output noise could be reduced by exceeding the limits of class-A_1 operation, perhaps because of grid-current limiting.

3. The conversion gain was observed not to increase greatly as the limits of class-A_1 operation were exceeded by the local-oscillator voltage excursions.

4. There were rare times when conversion gains considerably in excess of unity--of the order of four or five--were noted. The cause was not determined but could not be traced to the test equipment. Since theoretically the conversion gain is not confined to unity or less, but since experimentally the voltage gains on several different plate mixers have been observed usually to be less than unity and practically never much over unity, the seeming freak behavior may be worth further investigation. This point should be approached with

caution, however, because the possiblity of spurious causes was not eliminated.

5. For various plate mixers utilizing different types of tubes having amplification factors ranging from 3.0 to 85 but not having carefully controlled experimental conditions, the conversion gain appeared to vary but little, usually being between 0.6 and 0.8 or so.

6. Fourth-order intermodulation rejection estimated to be at least 95 db was regularly observed on one plate mixer employing a type 6J4 tube, this value having been obtained--perhaps accidentally--without any particular regard for the operating point. Cathode bias was employed, assisted by grid-leak bias in some cases. A question thus arises as to the virtues of self-biasing arrangements as opposed to fixed bias.

7. Some observations of uncertain significance or dependability, made earlier on a type 71-A tube, indicated the possibility that while intermodulation rejection might become poorer as the grid swing approached the grid-current region, the rejection might become somewhat better as the swing approached cutoff. These comparisons were made against the results obtained in limited-swing class-A_1 operation.

CONCLUSIONS

A triode, or three-element vacuum tube, may be employed in a plate-modulation type of circuit to provide a frequency changer having characteristics of rejection of envelope distortion, cross-modulation and intermodulation interference which are superior to those of more commonly used types of frequency changers. These superior characteristics are gained, however, at a sacrifice of conversion gain, by a factor about equal to the amplification factor, in comparison with the conversion gain which can be realized from the same tube in an equivalent grid-modulation type of circuit.

Called a "plate mixer," the device which utilizes the tube in the plate-modulation configuration has the local-oscillator voltage introduced into the grid-cathode circuit and the signal voltage introduced into the plate-cathode circuit. The intermediate-frequency output is taken from the plate-cathode circuit. The conversion gain of the class-A_1 plate mixer is theoretically independent of the tube amplification factor. The theoretical gain is only indirectly dependent upon the mutual conductance or its partial derivatives, being affected by ratios of these.

In the plate mixer operated as suggested, the importance of the fourth-degree curvature relative to the second-degree curvature in the transfer characteristic is independent of the applied-signal amplitude so far as the mixing action is concerned but not so far as the interference is concerned. The greater the amplitudes of the applied signals, the more rapidly is generated interference worsened.

The plate mixer is capable of handling signal voltages much larger, for the same relative-interference generation and without overloading, than can be handled by the same tube used in an equivalent grid-mixer circuit. The ratio of the signal-handling capabilities for the two comparative cases is approximately equal to the tube amplification factor, μ, when class-A_1 operation, which is recommended, is used.

Conversely, for the same signal voltage applied to the class-A_1 plate mixer and to the equivalent grid mixer, the plate mixer offers a decided improvement in distortion and interference rejection. The ratio of improvement is a function of a positive integral power of the amplification factor. For fourth-degree interference this ratio is proportional to μ^2. For higher degrees the corresponding exponent of μ increases in proportion to the degree.

One approach to the design of a plate mixer is the consideration of the device as a maximum-voltage generator of intermediate-frequency output. The tube type to be chosen for the purpose should not only be of design suitable for the radio-frequency range of the input signal, but also should have a large amplification factor and a large grid base and hence should be a high-plate-voltage type.

The nature of the plate-load impedance is a major factor in the design. For the maximum-voltage design the plate load should be resistive, having a value which, by comparison with the tube plate resistance, is very small at the desired-signal input frequency, very large at the designated intermediate frequency, and as nearly zero as possible at all other frequencies including zero. Approximations to these conditions may be obtained by judicious use of tuned circuits or other networks.

The proper selection of an operating point is also a major design consideration where best interference-rejection characteristics are desired. This operating point should be selected finally by trial and adjustment to the point at which greatest intermodulation rejection is obtained. However, it is possible and practical to stipulate from theory the region in which the trials should be made.

First, the transfer characteristic chosen should be the one corresponding to the highest d-c plate voltage for which the tube can be operated in class A_1 and still remain within manufacturer's specifications. Second, a curve in rectangular coordinates should be carefully plotted displaying the second partial derivative of the mutual conductance, g_m, (with respect to absolute grid-cathode voltage) versus this grid-cathode voltage. The grid-bias value for which this curve passes through a positive maximum nearest the center of the class-A_1 region will theoretically result in zero production of fourth-degree intermodulation and should be near the bias value corresponding to lowest total fourth-order intermodulation.

For greatest conversion gain and greatest interference rejection, the local-oscillator voltage amplitude should be made as great as class-A_1 operation will permit. This voltage should be as stable in amplitude as possible in order to provide stable conversion gain. Also, this voltage should be as clean a sine wave and be as free of noise as it can practically be made. Theoretically, more stable conversion gain can be obtained by use of a value of local-oscillator voltage lower than the usual maximum stipulated above, but experiment has indicated that the best practical stability is obtainable at the maximum.

The foregoing design stipulations are all based upon the conditions that the radio frequencies are much higher than the intermediate frequencies, that the intermediate frequencies are much higher than the frequencies of the original modulation, and that the local-oscillator frequency is sufficiently removed from the signal frequencies that the two can be separated by ordinary filtering.

RECOMMENDATIONS

As a matter of both academic and practical interest, further study is recommended on the following facets of the general subject of plate mixers.

1. The plate mixer as a device for producing maximum i-f power output.

2. Operation of the plate mixer beyond the limits of class-A_1 operation.

3. Operation of the plate mixer in the vicinities of cutoff and of the practical beginning of grid conduction.

4. Noise generation in the plate mixer.

5. The effects of noise applied by the local oscillator, both within and without the limits of class-A_1 operation of the mixer.

6. Possibilities for obtaining increased conversion gain without large sacrifices in rejection characteristics.

7. Effects and applications of various kinds of feedback in the plate mixer.

8. Specific reasons why the conversion gain of the class-A_1 triode plate mixer seems almost independent of the type of tube used.

9. Operation of the plate mixer with self-biasing arrangements.

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